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Parametric Rolling at Main Resonance

Original scientific paper

The parametric resonance of the induced rolling is a typical dangerous situation for the transversal stability of ships operating in longitudinal waves. This paper presents the results of experimental tests regarding the seakeeping performances of a 2700 dwt cargo model at full loading, in regular longitudinal waves. The induced roll motion, in the second instability domain, both at zero and design speed was observed. The conditions for occurrence of the parametric resonance were analysed and the instability domains of motion were identified. The theoretical analysis of the heave and pitch motions in longitudinal waves, in frequency domain, was performed by using a computer code based on the Frank-close fit method. A satisfactory correlation between the theoretical and experimental results was found for heave and pitch motions. In order to simulate the induced rolling a typical differential coupled equations system for heave, induced roll and pitch motions was used. The numerical solution was obtained using the Runge-Kutta method. The simulation results for the ship motions at zero speed, in following regular waves, are presented. A good agreement was obtained between the numerical and experimental results in time domain.

Keywords: *experimental tests, parametric rolling, time domain simulation*

Parametersko ljuljanje pri glavnoj rezonanciji

Izvorni znanstveni rad

Parametarska rezonancija ljuljanja tipična je opasnost za poprečni stabilitet broda na uzdužnim valovima. Članak prikazuje rezultate pokusa ponašanja 2700 dwt modela teretnog broda pod punim opterećenjem na pravilnim uzdužnim valovima. Opažene su pojave induciranog ljuljanja u drugoj domeni nestabilnosti pri nultoj i projektnoj brzini. Analizirani su uvjeti za pojavu parametarske rezonancije te su određena područja nestabilnosti njihanja. Teorijska analiza u frekventnoj domeni poniranja i posrtanja na uzdužnim valovima je provedena na računalu primjenom Frankove metode prilagođavanja. Za poniranja i posrtanje teorijski i eksperimentalni rezultati su u suglasju. Za numeričko rješenje korišten je postupak Runge-Kutta. Prikazani su rezultati njihanja broda na harmonijskim valovima pri nultoj brzini. U vremenskoj se domeni rezultati pokusa i numerički rezultati dobro slažu.

Ključne riječi: *parametarsko ljuljanja, pokusi, simulacija u vremenskoj domeni*

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1 Introduction

When the ship runs in longitudinal waves, the representative ship motions are the heave and pitch ones. Due to the ship-waves interaction the transversal metacentre position has a dynamic modification in time domain, the excitation period being equal to the incident wave one [1]. As an effect of lateral perturbations, an induced roll motion may occur. The amplitude of the roll motion can increase, in parametric resonance conditions, according to the relation

$$\frac{T_e}{T_\phi} = \frac{n}{2} \qquad (1)$$

where, T_e is the incident wave period and T_ϕ the roll natural period.

The roll parametric resonance is known as a typical dangerous situation for the transversal stability of ships operating in longitudinal waves.

The main causes of the induced roll motion are considered to be:

- the energetic "saturation" phenomena of heave and pitch motion [2];
- the nonlinear coupling of the heave or pitch motions with the induced roll one [3].

Nayfeh considers that the energy put into the pitch and heave motions by the wave excitations may be partially transferred into the roll motions by means of nonlinear coupling among these modes; consequently roll motion can be indirectly excited. The ship will spontaneously develop large amplitude roll motions at parametric resonance.

The first instability domain (n=1) is the most dangerous one for the ship's transverse stability, but large amplitude roll motions can occur in the main resonance domain, when n=2. The seakeeping tests in longitudinal waves of a cargo ship model [4] revealed the occurrence of parametric rolling, both at zero and design speed. The induced roll motion has occurred in a narrow frequency domain of the incident waves. The body plan of the cargo ship is shown in Figure 1. The main particulars and characteristics of the ship and the scaled model, at full loading condition, are listed in Table 1.

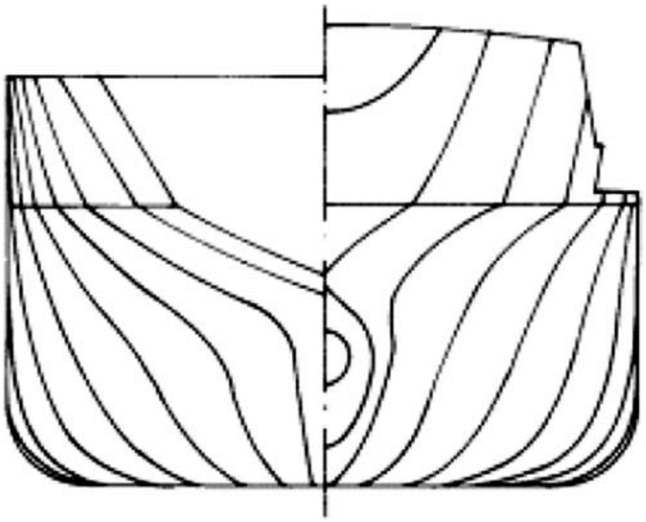


Figure 1 **The body plan of the cargo ship**
Slika 1 **Linije teretnog broda**

Table 1 **Main particulars at full loading condition**
Tablica 1 **Glavne izmjere za potpuno nakrcani brod**

Main characteristics	Full scale	Model scale (1/30)
Length over all, L_{max}	86.04 m	2.864 m
Length between perpendiculars, L	79.84 m	2.661 m
Breadth, B	14.5 m	0.483 m
Volumetric displacement, ∇	4297.0 m^3	0.159 m^3
Mean draught, T	5.245 m	0.175 m
Longitudinal centre of gravity, LCG	38.702 m	1.290 m
Vertical centre of gravity, KG	4.5 m	0.150 m
Metacentric height, GM_T	1.6 m	0.053 m
Natural period of roll motion, T_ϕ	7.55 s	1.38 s
Roll radius of gyration, k_{xx}	4.331 m	0.144 m
Pitch radius of gyration, k_{yy}	18.703 m	0.623 m
Yaw radius of gyration, k_{zz}	18.042 m	0.601 m
Ship speed, U	6.95 m/s	1.27 m/s
Froude number, F_n	0.25	0.25

2 Instability Domains of Induced Roll Motions

The uncoupled equation of the induced roll motion in longitudinal regular waves may be written using the Mathieu formulation [2]

$$(I_4 + A_{44})\ddot{\phi} + D(\phi,\dot{\phi}) + M_r(\phi,t) = 0 \tag{2}$$

where, I_4 is the inertia moment of roll motion, A_{44} is the added mass of roll motion, $D(\phi, \dot{\phi})$ is the roll damping moment, $M_r(\phi, t)$ is the restoring moment in longitudinal waves, ϕ is the roll angle and t is the time variable.

The dependency of the damping moment on the roll angle may be synthetically written as

$$D(\phi,\dot{\phi}) = \sum_{i,j} D_{ij}\phi^i\dot{\phi}^j, \text{ (i, j=0,1,2,...)} \tag{3}$$

where D_{ij} represents the damping coefficients.

Within the limits of a cubic approximation, the restoring moment may be written as

$$M_r(\phi,t) = gm[GM(t)\phi + k_3\phi^3] \tag{4}$$

where, $GM(t)$ is the time-dependent metacentric height, m is the ship mass and g is the gravitational acceleration.

When the ship runs in longitudinal waves, the metacentric height has a time-dependent harmonic variation expressed as follows

$$GM(t) = GM_m - GM_a \cos\omega t \tag{5}$$

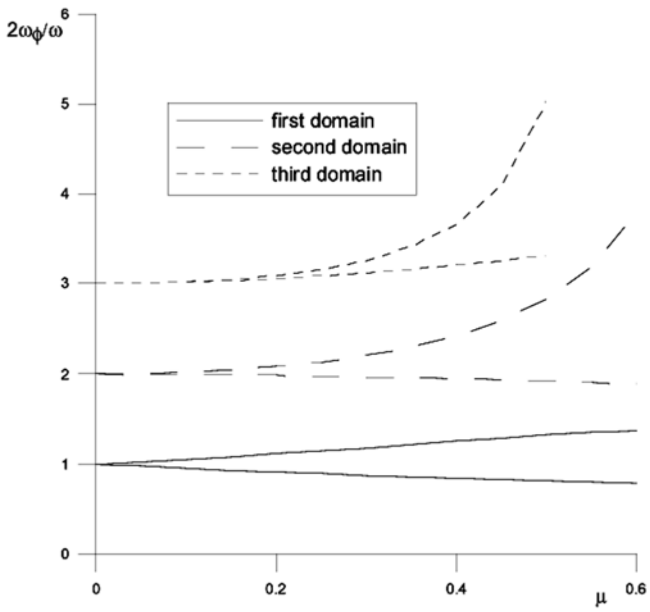
where, GM_m and GM_a are the mean value and the amplitude of the metacentric height and ω is the wave circular frequency. The excitation coefficient for the induced roll motion has the expression

$$\mu = 0.5GM_a / GM_m \tag{6}$$

The instability domains are corresponding to the critical frequency ones, where the ship may develop large amplitude roll motions.

A remarkable property of Mathieu's equation is that the solutions reach infinite values within instability domains. The frontiers of both stability and instability domains are defined by

Figure 2 **The instability domains of induced roll motion**
Slika 2 **Područje nestabilnosti induciranog ljuljanja**



T or 2T periodical solutions of the equation. By imposing Mathieu's equation to have 2T period solutions, the frontiers of the odd instability domain can be obtained. In the second instability domain, Mathieu's equation must have T period solution.

Figure 2 shows the diagram of the first three instability domains of the induced roll motion for the cargo ship. The vertical axis is the $2\omega_\phi/\omega$ ratio, where ω_ϕ, is the roll natural circular frequency and the horizontal axis is the excitation coefficient μ. One can observe that the first instability domain may lead to occurrence of the induced roll motion for small values of the excitation coefficient μ. The instability domains become larger for increased values of the excitation coefficient. The data from Figure 2 are obtained neglecting the damping moment contribution. If the influence of the damping term is considered, the area between the frontiers of the instability domains becomes smaller.

The excitation coefficient is not the only parameter that has an influence on induced roll motions. Another necessary condition may be mathematically written as follows

$$T_\phi = \frac{1}{n} \cdot \frac{4\pi}{\sqrt{\dfrac{2\pi g}{\lambda} - 2\pi \dfrac{U}{\lambda} \cos\alpha_i}}, \quad n \in N^* \tag{7}$$

Figure 3 **The necessary conditions for induced roll motions in the first instability domain**
Slika 3 **Potrebni uvjeti za induciranje ljuljanja u prvom području nestabilnosti**

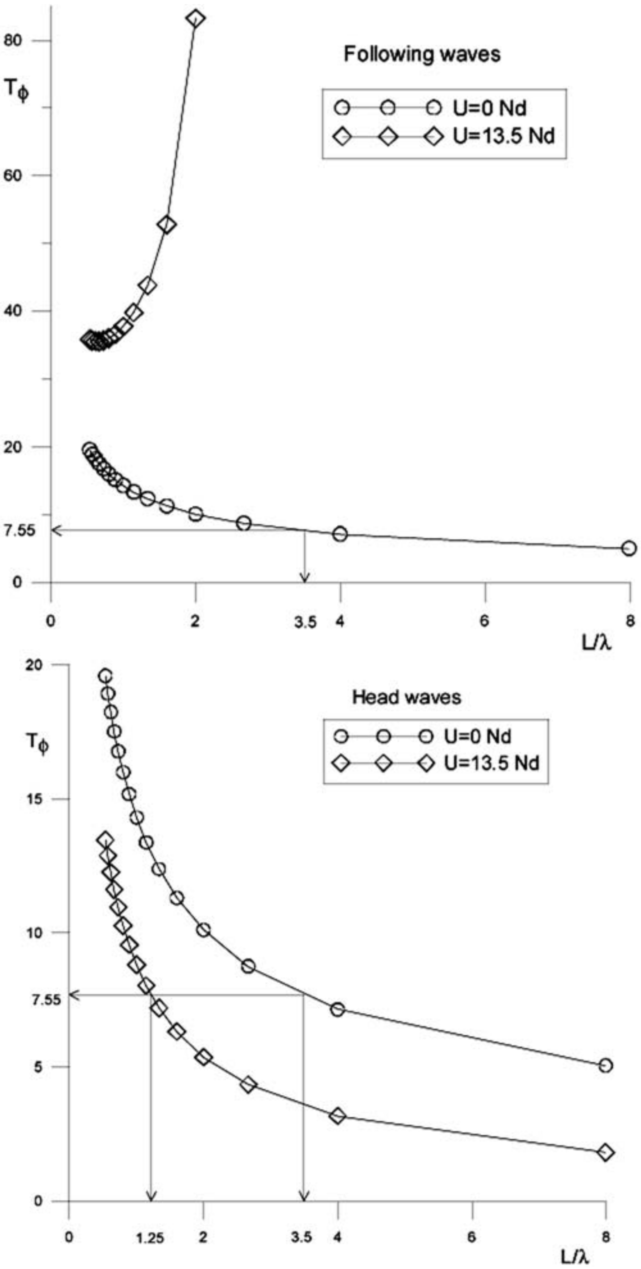
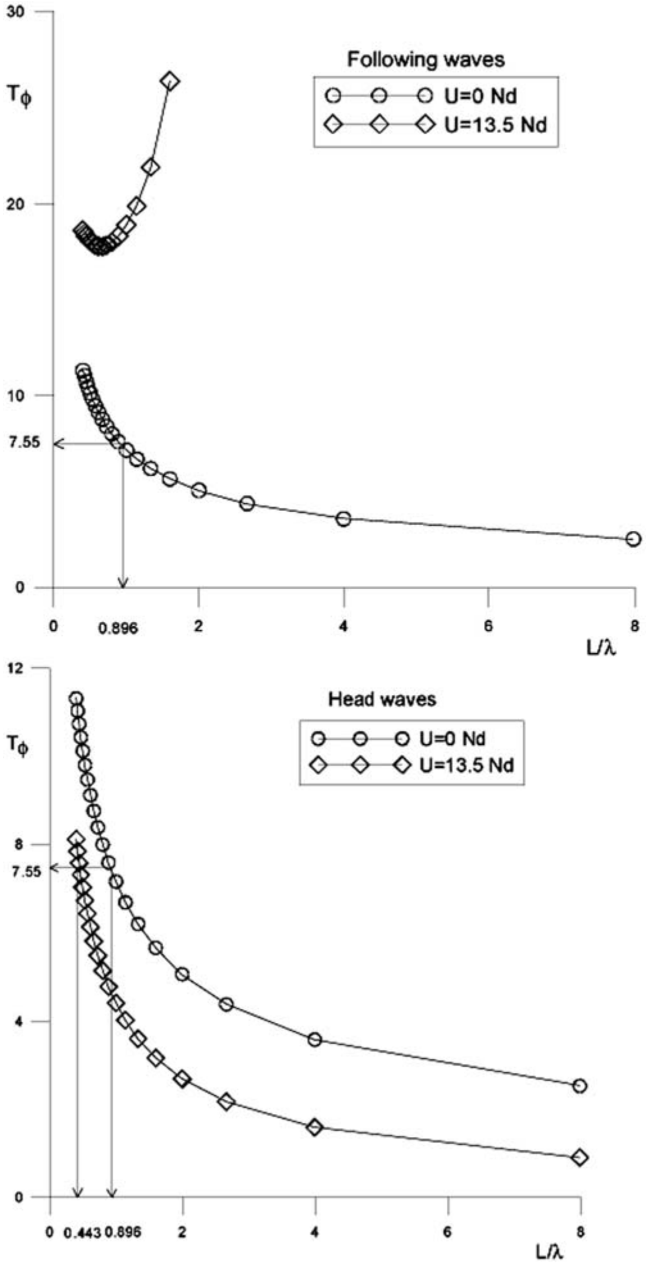


Figure 4 **The necessary conditions for induced roll motions in the second instability domain (main resonance)**
Slika 4 **Potrebni uvjeti za induciranje ljuljanja u drugom području nestabilnosti (glavna rezonancija)**



where, U is ship's speed and α_i is the heading angle. The graphical representation of the condition given by the equation (7) is shown in Figures 3 and 4. On the vertical axis the L/λ ratio is represented, where L is the ship length and λ is the length of the regular wave, and on the horizontal axis the natural roll period T_ϕ is represented. For the studied cargo ship, having the full scale natural roll period T_ϕ = 7.55 s, the probability of a roll motion occurrence in the second instability domain (main resonance) is increased. For the case at zero speed, the first instability domain corresponds to L/λ = 3.5 and the second instability domain to L/λ = 0.896. Regular waves, with small wavelength, do not have enough energy to generate induced roll motion. Consequently, at zero speed, parametric roll resonance in the first instability domain has low probability of occurrence. Moreover, due to the limitations of the wave generator, the experimental tests for f_m = 1.43 Hz (corresponding to L/λ = 3.5) could not be carried out. However, the induced roll motions at zero speed were measured, in the second instability domain, at main resonance, for both head and following waves respectively.

Figures 3 and 4 demonstrate that for both instability domains, the induced roll motion in regular following waves, at design speed, is not possible. When the ship runs at design speed in head waves, the probability of occurrence of roll motion in the second instability domain (L/λ = 0.443) is greater as compared to the first instability domain (L/λ = 1.25), where the amplitudes of motions of the ship model, on short waves, are very small. The main roll resonance for the ship running at design speed, in head waves, was observed and measured during the experimental tests. Comparing with the parametric resonance in the first instability domain, the induced roll motion at the main resonance for the ship in longitudinal waves was not intensively investigated. The literature offers little information on the presence of induced roll motion in the second instability domain [5]. From this point of view, a theoretical and experimental analysis of the main roll resonance represents a necessary extension of the studies on parametric roll resonance.

3 Theoretical and Experimental Determination of Ship Motions in Longitudinal Waves

The theoretical evaluation of ship motions in longitudinal waves has been obtained by using a computer code based on the Frank close-fit method [6]. Figures 5 and 6 show the transfer functions for heave and pitch motions respectively, in regular head waves, as a function of L/λ ratio. Figures 7 and 8 depict the transfer functions for heave and pitch motions respectively, in regular following waves, as a function of L/λ ratio. The non-dimensional formulations are obtained using z/ζ_w and $\theta/(k\zeta_w)$ ratios where, z and θ are the amplitudes of heave and pitch motions, ζ_w is the wave amplitude and k is the wave number.

The comparison between the theoretical and experimental results, for the heave and pitch motions, reveals a good agreement at zero speed. For the design speed the experimental transfer function are smaller than the theoretical predicted ones.

For the ship running in longitudinal waves, the computer code does not calculate a solution of induced roll motion. However, the experimental tests clearly demonstrate the existence of induced rolling at the main resonance. Figure 9 shows the experimental

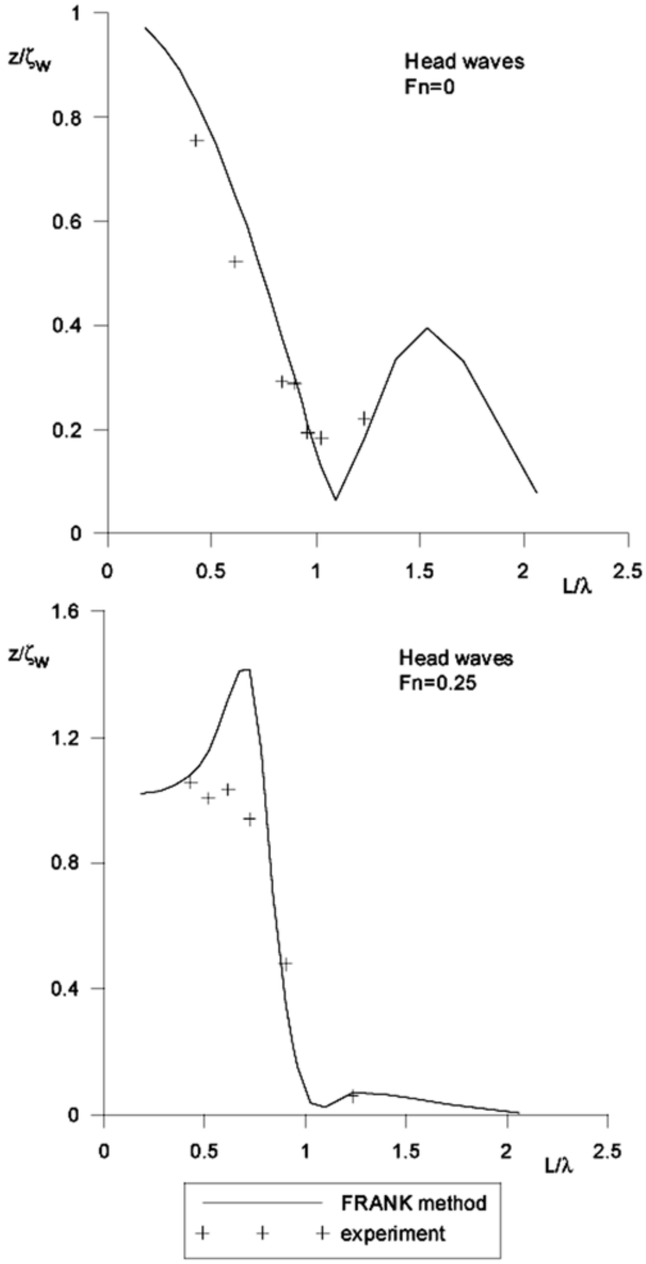


Figure 5 **Non-dimensional transfer functions of heave motion in regular head waves**
Slika 5 **Bezdimenzionalna prijenosna funkcija poniranja na pravilnim valovima u pramac**

values of non-dimensional transfer functions, $\phi/(k\zeta_w)$ depending on L/λ ratio, for induced roll motions in longitudinal waves.

The maximum amplitude is obtained at the main resonance, both at zero speed (L/λ = 0.896) and design speed (L/λ = 0.443). Figure 10 shows the experimental test arrangement in regular head waves.

The analysis of the experimental results given in Figure 9 leads to the following observations:

- the roll induced motions cover a small number of L/λ ratios, being a typical phenomena of narrow frequency domain;

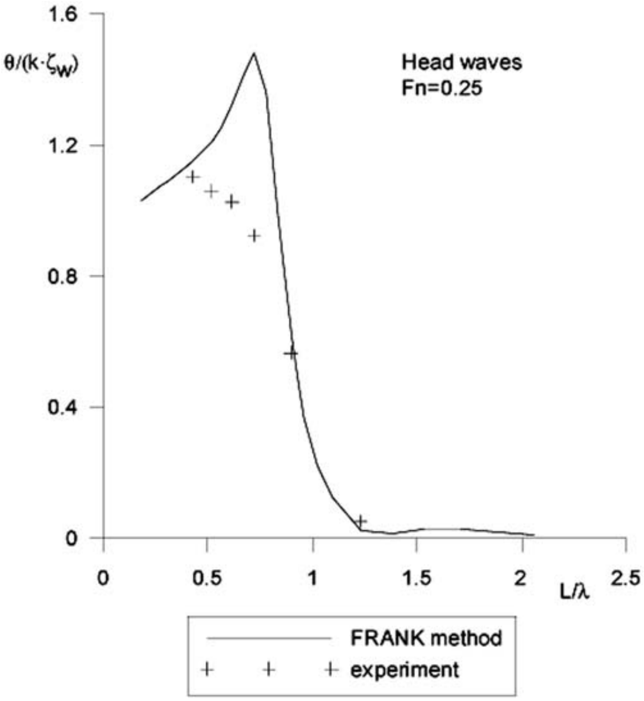


Figure 6 **Non-dimensional transfer functions of pitch motion in regular head waves**
Slika 6 **Bezdimenzionalna prijenosna funkcija posrtanja na pravilnim valovima u pramac**

- the amplitude of the induced roll motion at design speed is smaller than the amplitude at zero speed, in longitudinal waves;
- the model ship behaviour at zero speed in head waves, is similar to the following waves case;
- the coupled induced roll, heave and pitch motions are obtained in the second instability domain.

It is obvious that the development of the theoretical model must consider the induced rolling at the main resonance as being coupled with heave and pitch motions [7].

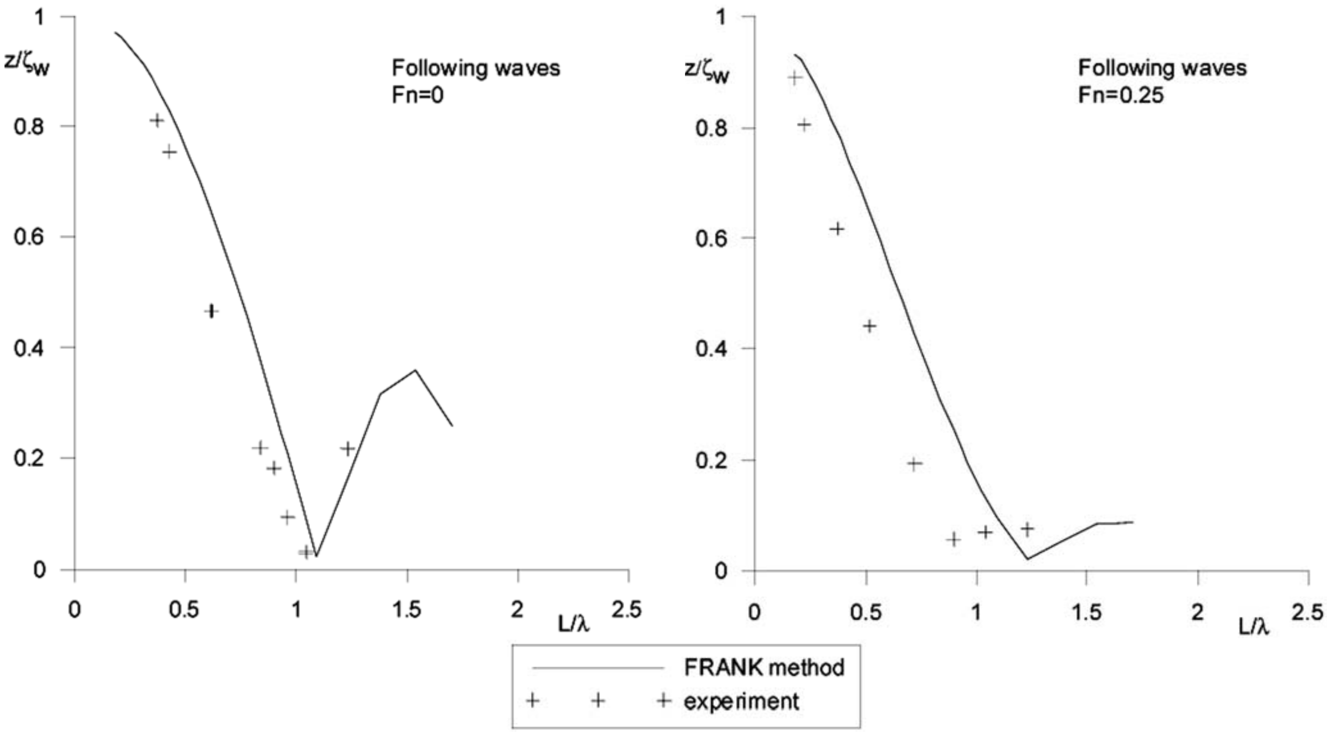
4 Numerical Modelling of Coupled Induced Roll, Heave and Pitch Motions

Considering the ship moving in regular longitudinal waves, heave (z), induced rolling (ϕ) and pitch (θ) motions were determined by numerical solving of the following coupled differential equation system

$$\begin{aligned} &(m + A_{33})\ddot{z} + A_{34}\cdot\ddot{\phi} + A_{35}\cdot\ddot{\theta} = q_1 \\ &A_{34}\cdot\ddot{z} + (I_4 + A_{44})\ddot{\phi} + A_{45}\cdot\ddot{\theta} = q_2 \\ &A_{35}\cdot\ddot{z} + A_{45}\cdot\ddot{\phi} + (I_5 + A_{55})\ddot{\theta} = q_3 \end{aligned} \tag{8}$$

$$\begin{aligned} &q_1 = F_{3a}\cdot\cos\omega t - (B_{33}\cdot\dot{z} + C_{33}\cdot z + \\ &+B_{34}\cdot\dot{\varphi} + B_{35}\cdot\dot{\theta} + C_{35}\cdot\theta) \\ &q_2 = F_{4a}\cdot\cos\omega t - [B_{34}\cdot\dot{z} + (D_{01} + \\ &+D_{02}\cdot|\dot{\varphi}|)\dot{\varphi} + g\cdot m(GM + \\ &+K_3\cdot\varphi^2)\varphi + B_{45}\cdot\dot{\theta}] \\ &q_3 = F_{5a}\cdot\cos\omega t - (B_{35}\cdot\dot{z} + C_{35}\cdot z + \\ &+B_{45}\cdot\dot{\varphi} + B_{55}\cdot\dot{\theta} + C_{55}\cdot\theta) \end{aligned} \tag{9}$$

Figure 7 **Non-dimensional transfer functions of heave motion in regular following waves**
Slika 7 **Bezdimenzionalna prijenosna funkcija poniranja na pravilnim valovima u krmu**



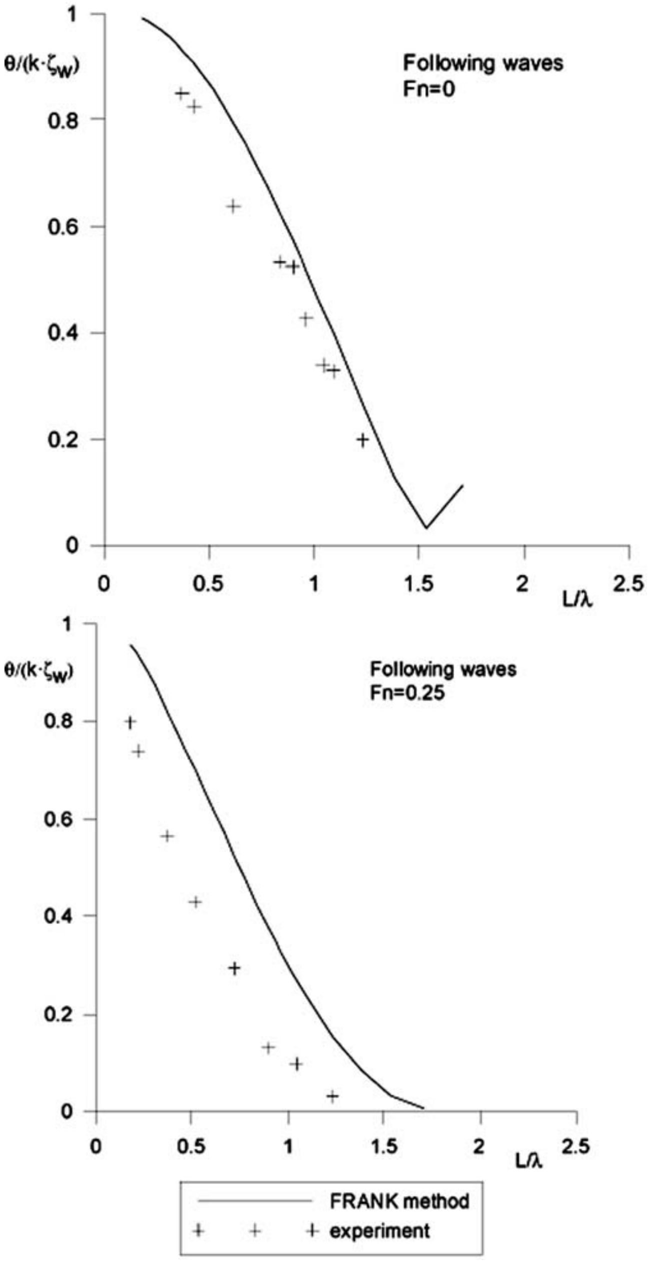


Figure 8 **Non-dimensional transfer functions of pitch motion in regular following waves**
Slika 8 **Bezdimenzionalna prijenosna funkcija posrtanja na pravilnim valovima u krmu**

A_{ij} and B_{ij} (i, j = 3, 4, 5) represent the added masses and the damping coefficients, respectively, C_{ij} are the restoring coefficients and F_{ia} (i=3, 4, 5) represent the amplitudes of excitation forces and moments generated by the incident waves.

The evaluation of A_{ij} components is based on theoretical and experimental approach of the radiation problem [8], where the coupled coefficients A_{34}, D_{34}, A_{45}, B_{45} are considered as linear functions of small heeling angle.

Solving the diffraction problem, the amplitudes of F_{ia} force were theoretically and experimentally determined [8]. Moreover,

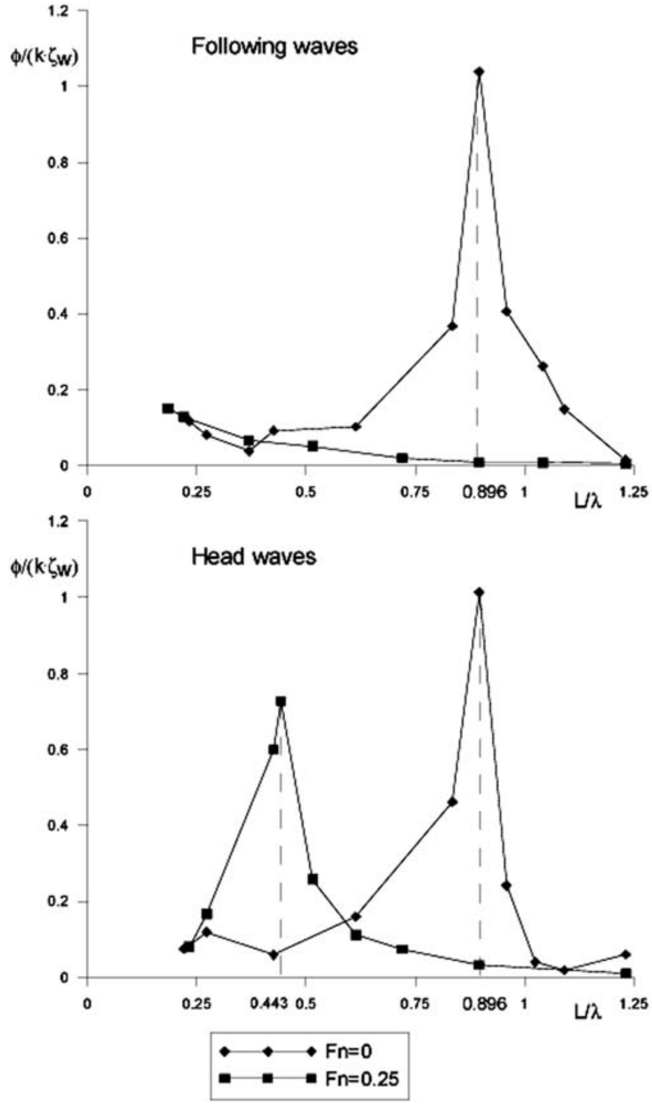
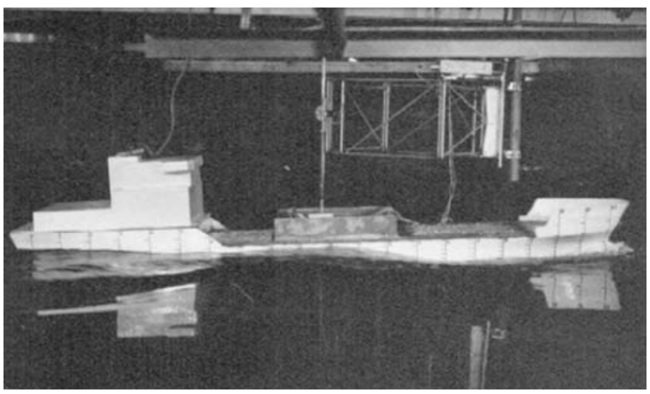


Figure 9 **Non-dimensional transfer functions of induced roll motion in regular longitudinal waves**
Slika 9 **Bezdimenzionalna prijenosna funkcija induciranog ljuljanja na pravilnim uzdužnim valovima**

Figure 10 **Experimental test in regular head waves**
Slika 10 **Pokusi na pravilnim valovima u pramac**



the amplitude of the excitation moment at induced rolling is a linear function depending on the heeling angle

$$F_{4a} = f_{4a}\phi \tag{10}$$

The damping coefficients of induced roll motion in longitudinal waves, D_{01} and D_{02} were estimated based on experimental roll decay tests [4], analysed through energetic method proposed by J.B. Roberts [9].

The hydrostatic and hydrodynamic values of the cargo ship model are presented in Table 2.

Table 2 **The hydrostatic and hydrodynamic values of the cargo ship model**
Tablica 2 **Hidrostatički i hidrodinamički podaci za teretni brod**

Symbol	Ship model values (model scale 1:30)
A_{33}	170.97 kg
B_{33}	682.3 kg/s
A_{55}	64.582 kg·m^2
B_{55}	271.632 kg·m^2/s
A_{44}	0.71 kg·m^2
D_{01}	0.7848 kg·m^2/s
D_{02}	0.00592 kg·m^2
A_{34}	3.6325 · $\lvert\phi\rvert$ kg·m
B_{34}	-13.853 · $\lvert\phi\rvert$ kg·m/s
A_{45}	-0.2913 · $\lvert\phi\rvert$ kg·m^2
B_{45}	-2.142 · $\lvert\phi\rvert$ kg·m^2/s
A_{35}	-4.965 kg·m
B_{35}	-30.082 kg·m/s
C_{33}	10592.65 kg/s^2
C_{44}	85.122 kg·m^2/s^2
C_{55}	4690.35 kg·m^2/s^2
C_{35}	26.13 kg·m/s^2
F_{3a}	33.42 kg·m/s^2
F_{4a}	36.95 · ϕ kg·m^2/s^2
F_{5a}	62.16 kg·m^2/s^2

The numerical solutions of the differential equation (8) were found using the Gill version of the 4[th] order Runge-Kutta method. The calculations were performed at zero speed, for the case of induced roll motions in regular following waves, at the corresponding values of $L/\lambda = 0.896$ and h_w/λ=1/50, where h_w is the wave height. These conditions are specific for the parametric resonance of the induced roll motions in the second instability domain (main resonance). The initial conditions used to solve the differential equations took into consideration the zero heeling angle case. The experimental tests carried out under the above mentioned conditions allowed measurement of heave, induced roll and pitch motions, having the following amplitudes: $z = 0.006\ m$, $\phi = 3.8^0$ and $\theta = 1.8^0$.

In Figure 11 the numerical and experimental results of the behaviour of the ship model at zero speed, in regular following waves at the main resonance condition are exemplified. The correlation between the numerical and experimental results is satisfactory. As compared to heave and pitch motions, which are reaching stabilised solutions for a small number of oscillations, the induced roll motion has firstly a transitory phase and then the rolling amplitude slowly increases and becomes close to the experimental measured value. The induced rolling amplitude depends on the unit amplitude of roll excitation moment f_{4a}, thus the excitation coefficient of the induced roll motion can be defined as

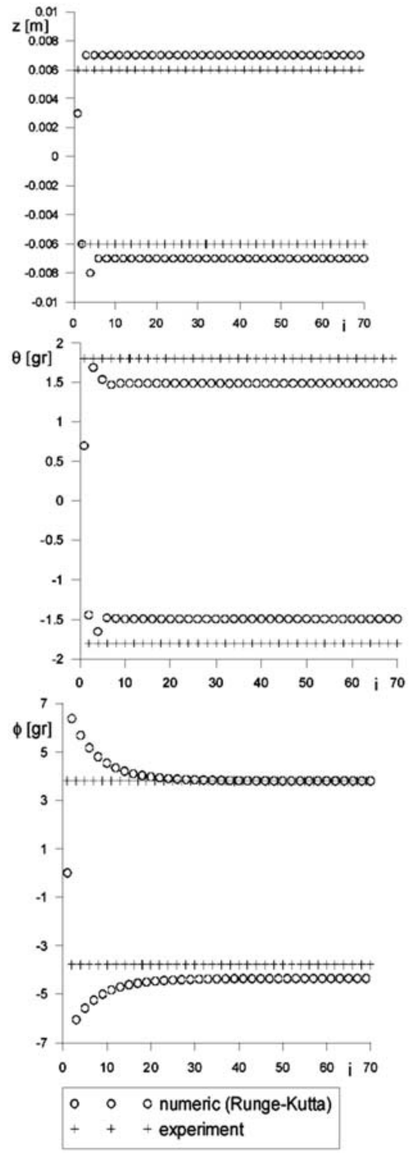
$$\mu = \frac{f_{4a}}{2 \cdot \omega_\phi^2 \cdot (I_4 + A_{44})} \tag{11}$$

where, ω_ϕ is the natural circular frequency.

5 Conclusions

The induced roll motion at parametric resonance represents a complex physical process. The occurrence of induced roll motions

Figure 11 **Numerical and experimental results of the behaviour of the ship model at zero speed, in regular following waves, at main resonance condition**
Slika 11 **Numerički i eksperimentalni rezultat ponašanja modela broda pri nultoj brzini na pravilnim valovima u uvjetima glavne rezonancije**



in the instability domains is a result of multiple interdependence of physical parameters such as: the energy partially transferred to the roll mode of motion, the excitation coefficient depending on the variation of the metacentric height on longitudinal waves, the natural roll period, the ship speed, the wave length, the heading angle, the encountering period, etc. Mention should be made that the diagrams given in Figures 3 and 4 suggest the correlation between parametric resonance condition (1) and the relation between the encountering period and the incident wave period. Based on the above mentioned diagrams and instability domains analysis, the physical conditions of the induced rolling may be identified.

The second instability domain was little analysed. This study was performed by coupling the differential equations of induced rolling with specific equations of heave and pitch motions. Except for the restoring coefficients and the coupled hydrodynamic coefficients, the remaining ones were experimentally measured during diffraction and radiation tests. The numerical solutions were experimentally validated.

Numerical tests were performed in order to identify the physical parameter having the main contribution on induced roll motion occurrence. It was demonstrated that the roll excitation moment had a decisive influence. In the considered case, a 10% reduction of roll excitation moment leads to the damping of induced roll motion. A 10% increment leads to a considerable amplification of ship's response.

The definition of the excitation coefficient, based on the experimental results of the diffraction problem, leads to a better understanding of the physical complex mechanism of the induced roll motions occurrence.

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