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Nonlinear Ship Rolling and Capsizing

Original scientific paper

The existing level of ship safety rules is presented and analysed, and guidelines for upgrading and improving the rules are given. An uncoupled equation of ship rolling is set and the methods for solving nonlinear ship rolling in regular and irregular waves are presented. Calculations of rolling for a particular ship in regular and irregular waves are done using the harmonic acceleration method. Nonlinear response phenomena are analysed. Capsizing probability is calculated by means of nonlinear dynamics (basin erosion technique). Based on these calculations new criteria for ship stability are recommended, based on the use of the ship survivability diagrams.

Keywords: *ship safety, stability, rolling, capsizing, nonlinear dynamics, chaos theory, ship survivability*

Nelinearno ljuljanje i prevrtanje broda

Izvorni znanstveni rad

U radu je dan prikaz postojeće razine sigurnosti brodova definirane pravilima, te smjernice moguće nadogradnje i poboljšanja pravila. Postavljena je raspregnuta jednadžba ljuljanja broda te je dan prikaz metoda za rješenje nelinearnog ljuljanja i prevrtanja broda na harmonijskom valu i morskim valovima. Proveden je proračun ljuljanja odabranog broda na harmonijskom valu i na morskim valovima primjenom metode harmonijskog ubrzanja. Analizirani su nelinearni fenomeni u odzivu. Proračun vjerojatnosti prevrtanja broda proveden je primjenom pristupa nelinearne dinamike (erozija bazena). Temeljem ovih izračuna predlažu se novi kriteriji o stabilitetu broda korištenjem dijagrama preživljavanja broda.

Ključne riječi: *sigurnost broda, stabilitet, ljuljanje, prevrtanje, nelinearna dinamika, teorija kaosa, sposobnost preživljavanja broda*

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1 Introduction

Safety against capsizing is of great importance for every ship. Capsizing could be regarded as a rare event, but the consequences of such event are fatal (loss of ship and the crew).

Practice shows that even ships which satisfy all the existing rules are exposed to the risk of capsizing. The existing level of ship safety is defined by numerous national and international rules. Tendencies for the improvement of ship safety are oriented towards:

- shift from deterministic towards probabilistic approach,
- uniting the knowledge of designers and direct experiences of ship crew members,
- taking into account specific characteristic of certain ship types,
- taking into account important influential factors missing from the existing rules.

Considering the fact that a ship besides demands on stability has to satisfy many other criteria, it is necessary to include real conditions a ship could be faced with, and that definitely includes probability in calculation.

A great effort is done to establish simple and available methods for the estimation of ship safety applicable during the design process and ship service. Suggestions concerning the criteria for and regulations on ship safety should satisfy: availability of input data, simplicity for application, reliability of the results and the possibility of the simple analysis of the results.

2 Review of ship capsizing cases

One of the most disastrous cases is definitely the capsizing of the passenger RO-RO ship *Estonia* in 1994 in the Baltic Sea which caused 852 human casualties. After the wave ripped off the bow gate, capsizing of the ship followed within only 15 minutes.

Table 1 **Accidents of merchant ships under the flag of Great Britain from 1994 to 2002, >100 gt**
Tablica 1 **Nesreće trgovačkih brodova pod zastavom Velike Britanije od 1994. do 2002., većih od 100 gt**

Year	1994	1995	1996	1997	1998	1999	2000	2001	2002
Capsizing	-	-	5	-	-	1	-	1	-
Total number of accidents	**249**	**236**	**251**	**237**	**217**	**159**	**141**	**133**	**122**

Table 2 **Accidents of fishing boats under the flag of Great Britain from 1994 to 2002**
Tablica 2 **Nesreće ribarskih brodova pod zastavom Velike Britanije od 1994. do 2002.**

Year	**1994**	**1995**	**1996**	**1997**	**1998**	**1999**	**2000**	**2001**	**2002**
Capsizing	9	7	9	8	11	15	4	3	5
Percentage of capsizing(%)	1.64	1.16	1.7	1.67	2.72	3.94	1.18	0.9	1.75
Total number of accidents	548	604	528	478	404	381	338	334	285

Statistical data about marine accidents according to the yearly reports of MAIB (Marine Accident Investigation Branch) [1] are shown in Tables 1 and 2.

Relevant statistical data about the accidents of ships under the flag of Japan according to the Japanese report on marine accidents [2] are shown in Table 3.

Table 3 **Accidents of ships under the flag of Japan during 2002**
Tablica 3 **Nesreće brodova pod zastavom Japana tijekom 2002.**

Ship type	**Number of capsized ships**	**Capsizing percentage (%)**	**Total number of ship accidents**
Passenger	1	2.70	37
Merchant	3	1.24	242
Pleasure boats	13	6.13	212
Fishing boats	10	2.00	502

3 Review of the existing rules about the ship safety

Besides the rules of the international organisations (IMO, SOLAS), the rules which define the ship safety in general (capsizing included) could be the national rules, the classification societies rules and the rules established by Port State Control. The international rules represent recommendations, and the national rules could make these recommendations obligatory.

The rules according to [3, 4] are divided in the following categories:

1. Deterministic rules – from empirical and statistical data (IMO Resolution A.167, 1968; based on Raholas work (1939)),
2. Probabilistic rules – based on probabilistic calculations (IMO Resolution A.562, 1985; based on Yamagatas work (1959)),
3. Performance based rules – based on model testing and numerical simulation (IMO SOLAS ’95 Resolution).

Practice showed the need for establishing the rules specialised for certain ship types [5]. The example of such rules is “Weather stability criterion for fishing boats – IMO Torremolinos Convention (for ships longer than 24 m)” from 1993.

Besides the rules a ship has to satisfy before delivery, instructions to the ship crew are of great importance. An example of such instructions is “Guidance to the Master for avoiding dangerous situations in following and quartering seas“(MSC/Circ.707) based on Takaishi work (1982) [6].

Practical stability criteria applied for ship design comprise static stability. The criterion of static stability is represented by the static stability arm moment curve which is used as the main estimator of ship's ability to resist capsizing. Besides this main requirement, there are additional requirements concerning the required amount of initial metacentric height, minimal amount of stability arm and heeling angle corresponding to the maximum stability arm. Dynamic stability criterion prescribes that the work of the restoring moment in the calm sea should be larger than the work of the heeling moment.

As obvious, for complicated dynamical situations static calculation is done. Only a nonlinear dynamical approach corresponds to the realistic situation.

An appropriate selection of external influences is also of great importance. Although the selection of maximum values would be on the safe side, it could greatly influence other important ship properties.

The existing criteria are mostly simple for use, but with an unknown level of safety. Criteria gained through statistical methods are no more reliable when changes and innovations appear in the design. Although it is considered that the weather criterion significantly improved the safety of ship service, certain items are becoming questionable when the range of applicability of the rules is extended outside the initial borders of the rules [3].

Criteria used in the design process and guidance for ship handling concerning ship stability in bad weather conditions are treated as separate and different issues. The designer should provide the following information: how to use the ship in the best way, how to identify and avoid the risk, how to mitigate the risk which is noticed. Such experience should contain two different types of information: change of risk at the sea as a function of ship's heading and speed and the instructions concerning when it is safe to conduct manoeuvre which could contain capsizing risk. The crew provides valuable information in this area, which could systematically improve the existing instructions.

The advance in numerical simulation and probabilistic analysis is enabling the development of instructions for ship handling based on risk.

Every change in rules according to IMO should provide the same average level of safety as the existing rules. The advance in IMO rules is slow with the average time interval of 20 to 30 years between the scientific approval and practical application. When the improvement of the rules is considered, according to [3] it is important to take into account: simplicity, economic implication, connection with engineering practice of ship design.

IMO has made the revision of the rules for stability of intact ship at SLF 45 (Sub-Committee on Stability, Load Lines and Fishing Vessels) in 2002 and SLF 46 in 2003. The conclusion is that some changes in the weather criterion and the inclusion in the rules so far excluded phenomena (wind action, manoeuvrabilty ...) are inevitable [3].

4 Ship rolling and capsizing

One of the main causes of ship capsizing in waves is the loss of stability at rolling. Ship resists to capsizing with her restoring moment and the capability of dissipation of energy via damping.

According to [7], the following modes of capsizing are known (division by ITTC):

- static loss of stability ("surf-riding")
- dynamic loss of stability
  - dynamic rolling
  - parametric excitation
  - resonant excitation
  - impact excitation
  - bifurcations
  - broaching.

If the rolling on beam waves is considered, it is possible to ignore the coupling of rolling and the other degrees of freedom of ship motion. Ship rolling in beam waves could be regarded as a simple pendulum. For solving the rolling problem the linear and nonlinear approach could be used. The linear approach is applied for small rolling amplitudes, and the problem is solved in the frequency domain by the spectral analysis.

Ship rolling with larger amplitudes (and such situations lead to capsizing) represents a nonlinear problem. In this case the nonlinear equation of rolling is set in order to foresee the ship's nonlinear response.

The equation of rolling could be set for regular or irregular waves. The problem of ship rolling could be analysed in the frequency and time domain or by applying the methods of dynamics of nonlinear systems. The analysis in the time domain is more acceptable for this problem than the analysis in the frequency domain. A disadvantage is the need for conducting a great number of realizations in order to determine the probability of capsizing.

For the analysis of dynamic behaviour of nonlinear systems i.e. nonlinear oscillations where ship rolling is included, the following methods are developed:

- perturbation method [8]
- method of multiple time scales [9]
- methods of time averaging [10]
- Krylov-Bogoliubov-Mitropolsky method [11]
- harmonic balance method [12]
- time integration by Runge-Kutta or harmonic acceleration method [13]
- Galerkin method [10].

Lately, more attention is given to the approach of the direct determination of the probability of capsizing without the repetition of random realizations in the time domain. The probabilistic approach demands solutions of numerous problems which include the calculation of capsizing in the given time interval. In this case, the description of the sea state is also probabilistic. According to [14], the methods which tackle the mentioned problems are as follows:

- analysis in phase plane [Sevastianov 1979, Umeda 1992]
- dynamic heel angle determination [Dudziak 1978, Bielanski 1994]
- stability motion determination [Price 1975]
- nonlinear dynamics methods (i.e. basin erosion method) [Hsieh 1994, Lin and Solomon 1995]
- discretised linear method [Belenky 1994]
- Markov process theory [Roberts 1982].

It is important to know the possible modes of capsizing and the interrelationships of certain influence parameters based on the results of model testing and numerical simulations. Researches using mathematical models in conjunction with the theoretical development in the dynamics of nonlinear systems lead to improved understanding and insight to the nature of the ship capsizing process.

5 Review of nonlinear phenomena

The main characteristic of nonlinear systems is the possibility of occurrence of significant changes in the behaviour of the systems due to small variations of one of the parameters. According to [15], ship rolling with monoharmonic excitation enables the insight in the series of the following nonlinear phenomena:

- shift of the natural frequency
- multiple responses with jumps
- symmetry breaking
- resonant phenomena: superharmonic, subharmonic, super-subharmonic response
- bifurcations
- transition to chaos through cascade
- fractal boundary
- chaotic attractor
- chaos.

The nonlinear system could have a polyharmonic irregular response to monoharmonic excitation which is called deterministic chaos. Considering the ship rolling in sea waves, transfer to chaos i.e. fractal boundary of stability area could be observed and the so called basin of erosion could be formed.

The aim is to establish which combinations of parameters (rolling angles, wave heights, ship heading, ship speed, draught and so on) can cause the ship capsizing.

For certain combinations of parameters it is desirable to determine the risk of capsizing in a fast and efficient way. Although the situation of monoharmonic excitation is not realistic, the development of nonlinear phenomena observed in this way could serve as a precursor of possible capsizing i.e. to point out situations for which the calculation with polyharmonic excitaton should be done [16].

6 Calculation procedure

According to [17,18], the nonlinear equation of rolling in general form reads

$$(I_{xx} + \delta I_{xx})\ddot{\theta} + D(\dot{\theta},\theta) + R(\theta) = M(t) \tag{6.1}$$

where

$(I_{xx} + \delta I_{xx})$ - virtual moment of inertia (sum of inertia moments of the ship mass and added mass of surrounding water)
$D(\dot{\theta}, \theta)$ - nonlinear damping moment
$R(\theta)$ - nonlinear restoring moment
$M(t)$ - outer excitation moment

To determine the value of the moment of inertia if the rolling period is known, the following expression according to [4] is used:

$$T = 2\pi \sqrt{\frac{I_{xx} + \delta I_{xx}}{\Delta g MG}} \tag{6.2}$$

in the form:

$$(I_{xx}+\delta I_{xx})=(\frac{T}{2\pi})^2\Delta gMG \qquad (6.3)$$

where

T - rolling period
Δ - displacement mass
MG - metacentric height.

According to [19], the total damping moment could be regarded as the sum of linear D_L and nonlinear D_N contribution

$$D=D_L(V,\omega)+D_N(V,a) \qquad (6.4)$$

where

V - speed
a - excitation amplitude
ω - frequency.

The damping moment could be represented in numerous ways, in this work the linear formulation and the formulation of the cubic polynomial are used

$$D(\dot\theta)=D_1\dot\theta+D_3\dot\theta^3 \qquad (6.5)$$

If the expression is divided by I, it is obtained

$$d(\dot\theta)=d_1\dot\theta+d_3\dot\theta^3 \qquad (6.6)$$

where d_1 and d_3 are the relative damping coefficients.

The restoring moment is function of the underwater form of the ship hull. According to [17] it is usually represented as the odd polynomial of roll angles (if the ship is symmetric and in upright position), such as

$$R(\theta)=K_1\theta+K_3\theta^3+K_5\theta^5+...+K_m\theta^m \qquad (6.7)$$

In this work, polynomials of 3rd, 5th and 7th degree are used in the form

$$r(\theta)=k_1\theta+k_3\theta^3+k_5\theta^5+...+k_m\theta^m \qquad (6.8)$$

where k_i are the relative restoring coefficients.

Coefficients of this polynomial are gained in such a way, that they approximate the curve of stability arm in the best possible way.

According to [20] for the differential equation of rolling in the case of a damaged ship it is suggested

$$\ddot\theta+D\,\dot\theta-R\theta(1-\alpha_D{}^2\theta^2)(1-\beta_D{}^2\theta^2)=M(t) \qquad (6.9)$$

where α_D and β_D are the coefficients which describe the reduction of the restoring moment.

The outer wave excitation could be approximated by a harmonic sinusoidal function, but the attention should be paid to the fact that this kind of approximation is not appropriate for realistic calculations

$$M(t)=a\cos\omega t \qquad (6.10)$$

According to [21], the expression for the amplitude reads

$$a=I\alpha_0\omega_0^2\pi\frac{H_W}{l_W} \qquad (6.11)$$

where

α_0 - effective wave slope
ω_0 - wave frequency
H_w - wave height
l_w - wave length.

If the ship speed V and heading angle χ are taken into account

$$M(t)=I\alpha_0\omega_0^2\pi\frac{H_W}{l_W}\sin\chi\cos(\omega_e t) \qquad (6.12)$$

where

$\omega_e=\omega-\dfrac{\omega^2 V}{g}\cos\chi$ - encounter frequency.

According to [21], the equation for polyharmonic excitation is given

$$M(t)=a\sum_{n=N_1}^{N_2}c_{S\text{n}}\cos(n\omega t+\varepsilon_\text{n}) \qquad (6.13)$$

where

c_S - spectral coefficient, normalized amplitude of the excitation harmonics
ε - random phase angle, probability of occurrence is equally divided in the range from 0 to 2π.

When the expression for $M(t)$ is divided by I, it is obtained

$$f(t)=\frac{M(t)}{I}$$

The final form of nonlinear equation of rolling yields

$$\ddot\theta+\sum_{i=1}^{n}d_i\dot\theta^i+\sum_{i=1}^{n}k_i\theta^i=f(t) \qquad (6.14)$$

The equation is solved by the harmonic acceleration method.

According to this method [17], the equation of rolling is presented in the form

$$\ddot\theta+2\xi\omega\dot\theta+\omega^2\theta=\psi(t,\theta,\dot\theta) \qquad (6.15)$$

where

ξ - nondimensional damping coefficient

The right part of the equation is represented as the so called pseudoexcitation

$$\psi(t)=f(t)+(2\xi\omega)\dot\theta-\sum_{i=1,3}d_i\dot\theta^i+\omega^2\theta-\sum_{i=1,3,5}k_i\theta^i \qquad (6.16)$$

The expression for the acceleration is represented in a harmonic form

$$\ddot\theta_i=u\cos\omega_i t+v\sin\omega_i t \qquad (6.17)$$

where u and v are unknowns.

According to [17], the equation is solved using the following algorithm

$$\{Y\}_{i+1}^{k+1} = [U]\{Y\}_i + \{O\}\Psi_{i+1}^{k} \tag{6.18}$$

where

$\{Y\}$ - response vector (displacement, velocity and acceleration)
$\{O\}$ - loading vector
$[U]$ - transfer matrix.

Coefficients in this expression depend on the frequency, time step and nondimensional damping [13,16,17].

It is necessary to set the initial conditions in the form

$$\theta(0) = \theta_0 \tag{6.19}$$

$$\dot{\theta}(0) = \dot{\theta}_0 \tag{6.20}$$

The solution of the equation gives the rolling angles in the time domain. The excitation values are chosen and time integration is performed. Estimation of the probability of ship capsizing for certain outer conditions and initial conditions is conducted using the method of basin erosion. For this purpose the figure of the discretised phase space is determined [22]. This area is divided into parts.

The trajectory of each point is calculated until the time moment in which the decision whether the point belongs to the basin is made. Erosion coefficient is number of cases (with specified initial conditions) which remain in the basin after a certain number of iterations.

7 Illustrative example

For the calculation of ship rolling the form of the ferry (see Table 4 and Figure 1) tested in Brodarski Institute in 1997 is chosen [23]. The model testing results enable a comparison with the results gained numerically, and thus also validation of the applied procedure.

Table 4 **Characteristics of the analysed ship**
Tablica 4 **Značajke analiziranog broda**

Ship type : **FERRY FOR VEHICLES AND PASSENGERS**		
Length between perpendiculars/ Breadth	L_{pp}/B	3.656
Length on waterline/breadth	L_{WL}/B	3.632
Breadth/Draught	B/d	4.182
Longitudinal position of the centre of buoyancy	LCB	51% L_{pp}
Vertical position of the centre of buoyancy	ZCB	56.7%d
Difference of vertical positions of the centre of buoyancy and centre of gravitation	ZCG-ZCB	3.303 m
Waterline coefficient	C_{WL}	0.8107
Maximum section coefficient	C_M	0.9279
Block coefficient	$C_B\,(L_{WL})$	0.603

The ship is tested with and without the bilge keels and the calculations are done for both variants.

The model is manufactured in a scale Λ=15 (1:15).

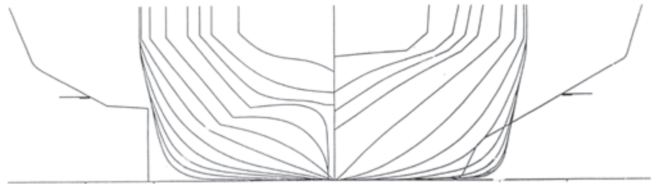


Figure 1 **Theoretical frames, bow and stern contour**
Slika 1 **Teoretska rebra, pramčana i krmena kontura**

7.1 Coefficients in the equation of rolling

The virtual moment of inertia is defined according to the expression (6.3)

I = 29682 tm^2 – ship without the bilge keels
I = 34303 tm^2 - ship with bilge keels

Coefficients of the restoring moment are obtained in a way presented in Chapter 6. The results are shown at Figures 2 and 3 and Tables 5 and 6.

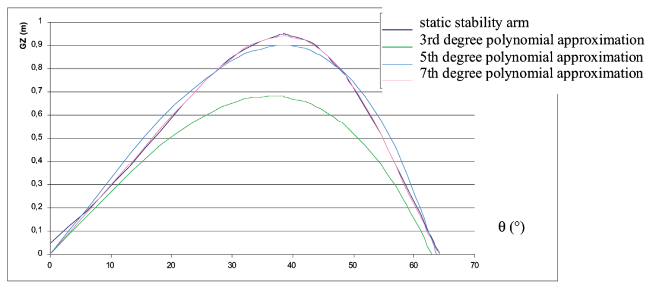


Figure 2 **Static stability diagram for the intact ship and its approximations**
Slika 2 **Dijagram statičkog stabilititeta i njegove aproksimacije**

Table 5 **Coefficients of the relative restoring moment for the intact ship**
Tablica 5 **Koeficijenti relativnog povratnog momenta za neoštećeni brod**

	k_1 (s^{-2})	k_3 (s^{-2})	k_5 (s^{-2})	k_7 (s^{-2})
3rd degree polynomial	0.671997033	-0.53920393	0.0	0.0
5th degree polynomial	0.630620936	-0.40482026	-0.08679720	0.0
7th degree polynomial	0.536133048	0.21992815	-1.128716074	0.4885682579

Figure 3 **Comparison of the static stability curves for the intact and damaged ship**
Slika 3 **Usporedba krivulja statičkog stabiliteta za oštećeni i neoštećeni brod**

The coefficients of the relative restoring moments for the damaged ship are given in Table 6.

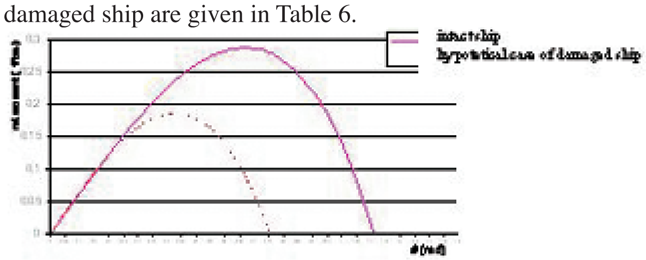


Table 6 **Coefficients of the relative restoring moment for the damaged ship**
Tablica 6 **Koeficijenti relativnog povratnog momenta za oštećeni brod**

	k_1 (s^{-2})	k_3 (s^{-2})	k_5 (s^{-2})
5th degree polynomial	0.65	-1.2	0.105

The linear coefficients of damping are determined on the basis of the roll decay test, see Figures 4 and 5, and the cubic coefficient is calculated in such way that the solution of the equation of rolling with the method of harmonic acceleration gives the best approximation (using the method of least squares).

Ship without bilge keels

d_1= 0.01265913 s^{-1}
d_3= 0.4954 s

Ship with bilge keels

d_1=0.0476542 s^{-1}
d_3=3.765 s

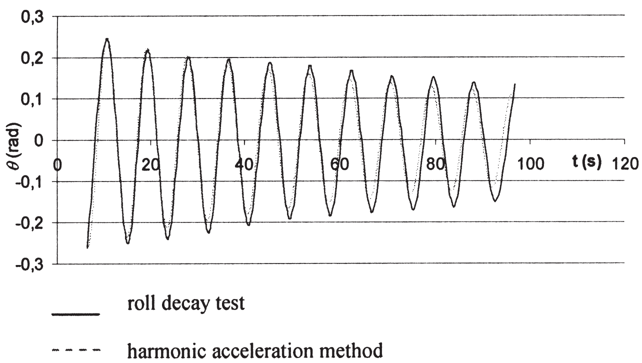


Figure 4 **Roll decay diagram - intact ship**
Slika 4 **Dijagram stišavanja ljuljanja – neoštećeni brod**

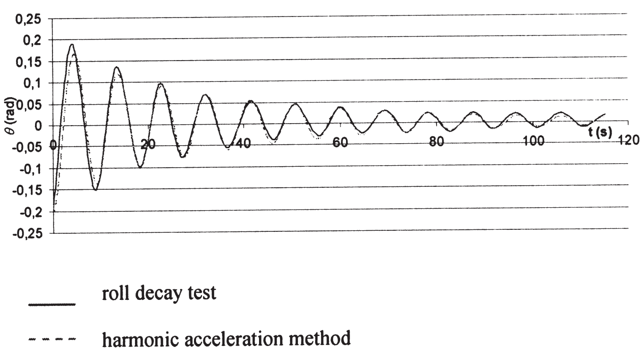


Figure 5 **Roll decay diagram - intact ship with bilge keels**
Slika 5 **Dijagram stišavanja ljuljanja za neoštećeni brod s ljuljnim kobilicama**

The monoharmonic sinusoidal excitation and the polyharmonic excitation describing the Adriatic Sea spectrum are used according to [24].

7.2 Comparison of the experimental results and numerical simulation by the method of harmonic acceleration

Comparison of the results obtained by the model testing and by the harmonic acceleration method is done for the following values of excitation

$H_{1/3}$= 3.75 m – significant wave height
χ = 90^0 (beam waves)
V_{ship} = 0.0 m/s

The effective wave slope was taken for all calculated conditions as follows

$$r=1.15123$$

For the numerical integration with the harmonic acceleration method the following parameters are used

T = 502.65 s - time period equal to the time period of the lowest harmonic
δt = 0.06135 s - time step

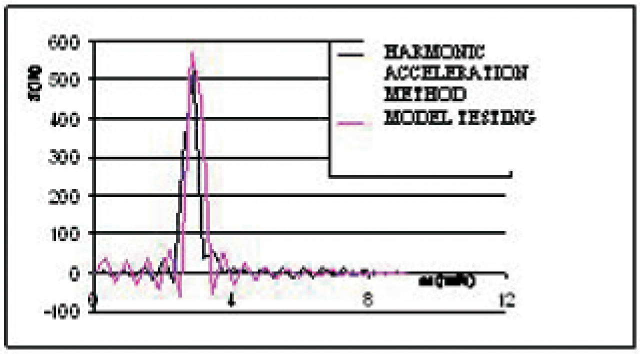


Figure 6 **Response spectra comparison for the intact ship**
Slika 6 **Usporedba spektara odziva za neoštećeni brod**

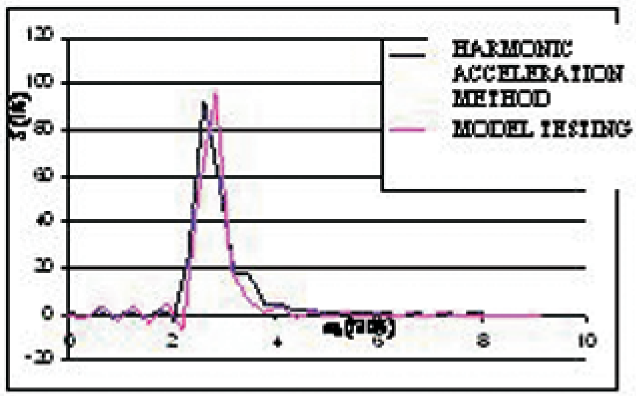


Figure 7 **Response spectra comparison for the intact ship with bilge keels**
Slika 7 **Usporedba spektara odziva za neoštećeni brod s ljuljnim kobilicama**

Comparison of spectra (Figures 6 and 7) points to satisfying correlation of the experimentally and numerically obtained results, i.e. simmilarity of spectra values.

7.3 Calculation of rolling in regular waves

For the calculation of rolling and capsizing of the intact ship in harmonic wave, the equation of rolling in the so called Duffing form is used (linear damping, nonlinear restoring moment):

$$\ddot{\theta} + d_1\dot{\theta} + k_1\theta - k_3\theta^3 = a\cos\omega t \qquad (7.1)$$

and for the calculations for the damaged ship, the equation in the following form is used

$$\ddot{\theta} + d_1\dot{\theta} + k_1\theta + k_3\theta^3 + k_5\theta^5 = a\cos\omega t \qquad (7.2)$$

The values of excitation parameters are used in the way that the results manifest the nonlinear phenomena. Stability of the solution is proved by application of Floquet's theorem [25].

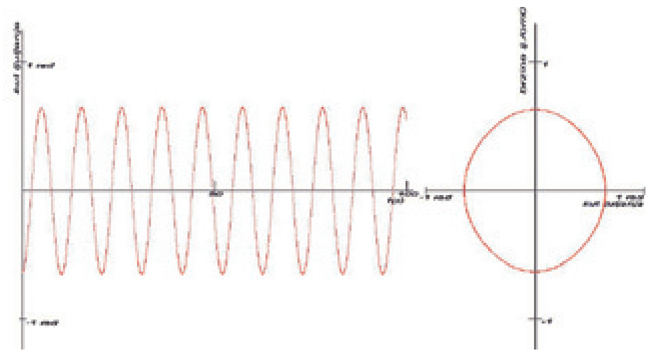


Figure 8 **Regular harmonic response to the harmonic excitation**
Slika 8 **Pravilan odziv na harmonijsku uzbudu**

$\dfrac{\omega}{\omega_0} = 0.915$, $\dfrac{a_1}{\omega_0^2} = 0.2666$, $\theta_{max} = 0.6449$ rad

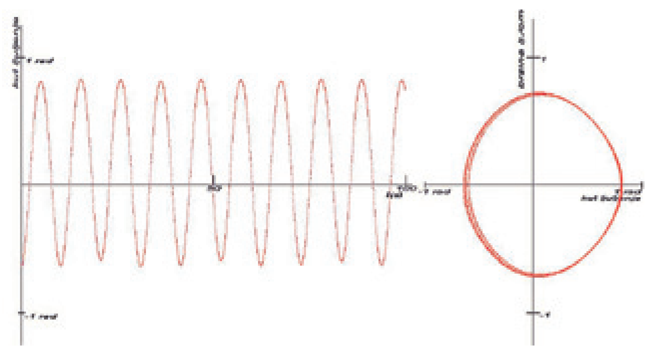
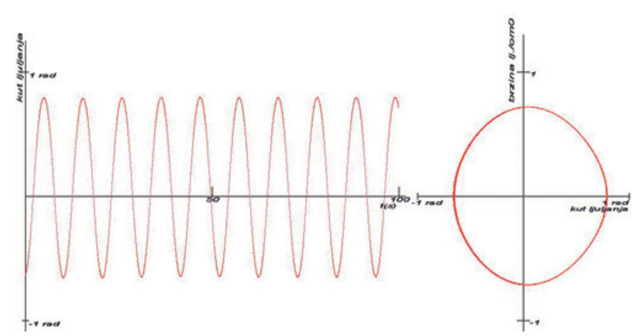


Figure 9 **Symmetry breaking**
Slika 9 **Poremećaj simetrije**

$\dfrac{\omega}{\omega_0} = 0.915$, $\dfrac{a_1}{\omega_0^2} = 0.3755$, positive amplitude $\theta_{max} = 0.7916$ rad, negative amplitude $\theta_{min} = -0.659$ rad

Figure 10 **Period doubling**
Slika 10 **Udvostručenje perioda**

$\dfrac{\omega}{\omega_0} = 0.915$, $\dfrac{a_1}{\omega_0^2} = 0.3848$



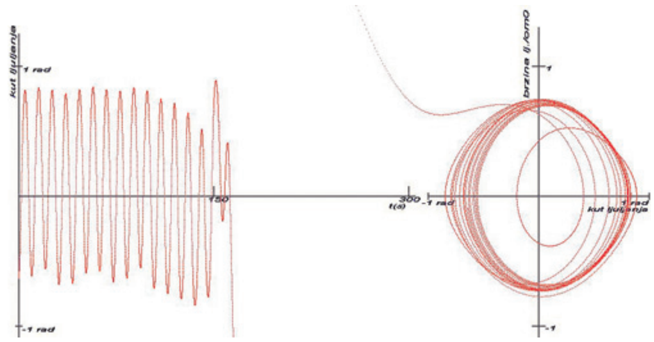


Figure 11 **Multiple periods, chaos and capsizing**
Slika 11 **Višestruki periodi, kaos i prevrtanje**

$\dfrac{\omega}{\omega_0} = 0.915$, $\dfrac{a_1}{\omega_0^2} = 0.388$

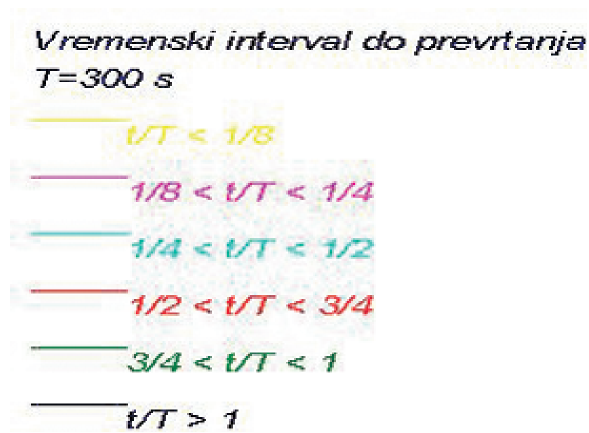
Based on this example, it is possible to follow the development of nonlinear phenomena which lead to the chaotic response and finally to ship capsizing.

First, regular harmonic response to the harmonic excitation is presented, see Figure 8.

By increasing the amplitude of excitation the symmetry breaking, i.e. the occurrence of changes of the values of positive amplitudes related to negative values occurs, see Figure 9. After that, the period doubling, i.e. occurrence of two different periods happens, see Figure 10. After that, multiple periods, chaotic motions, and finally capsizing occur, see Figure 11. Very small changes of the excitation amplitude values could cause such changes in the response.

Results of the repeated numerical calculations could be represented in the initial values plane (excitation values kept constant, initial values variable) or in the excitation plane (initial values constant, excitation amplitude and frequency variable). The calculation for each point is done in a certain time period. If during this period capsizing occurs, the point is drawn in a different colour (depends on the time period to capsize, see Fig.12 for legend of the colours). If capsizing does not occur, the point is drawn in blue. All the points which represent the situation when the capsize does not occur form the so called "safe basin".

Figure 12 **Legend for values in Figures 13-27**
Slika 12 **Legenda za vrijednosti na slikama 13-27**



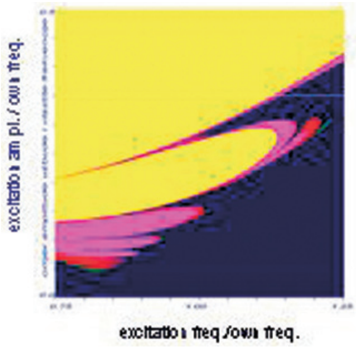


Figure 13 **Safe basin in the excitation plane, intact ship**
Slika 13 **Bazen sigurnosti u ravnini uzbude, neoštećeni brod**

$\theta = 0,\ \dot{\theta} = 0$

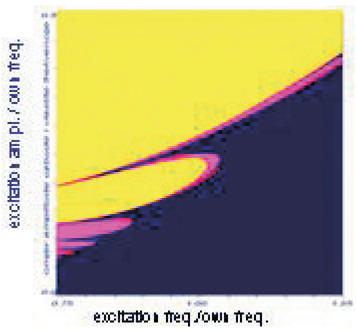


Figure 14 **Safe basin in the excitation plane, intact ship with bilge keels,**
Slika 14 **Bazen sigurnosti u ravnini uzbude, neoštećeni brod s ljuljnim kobilicama**

$\theta = 0,\ \dot{\theta} = 0$

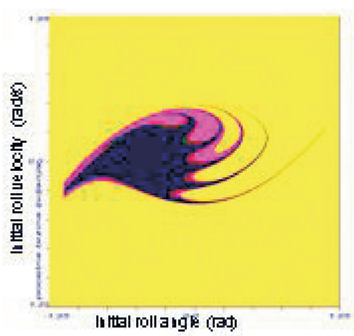
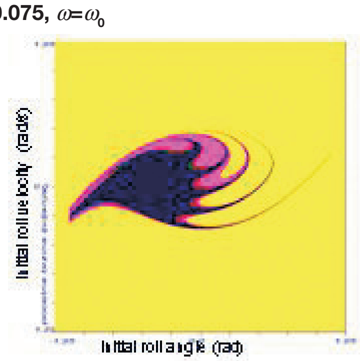


Figure 15 **Safe basin in the initial value plane, intact ship**
Slika 15 **Bazen sigurnosti u ravnini početnih uvjeta, neoštećeni brod**

a=0.075, $\omega=\omega_0$



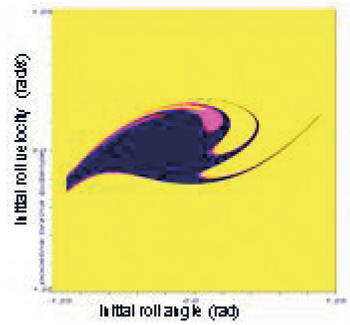


Figure 16 **Safe basin in the initial value plane, intact ship with bilge keels**
Slika 16 **Bazen sigurnosti u ravnini početnih uvjeta, neoštećeni brod s ljuljnim kobilicama**

a=0.21705, $\omega=\omega_0$

In Figures 13-16 safe basins in excitation and initial value plane for the intact ship with and without bilge keels are shown.

The safe basin of the intact ship in the excitation plane shows the significant influence of the bilge keels. The capsizing area for ships with bilge keels is much smaller and the transition to capsizing condition is much sharper. For the ship without the bilge keels this transition is of extensively fractal nature.

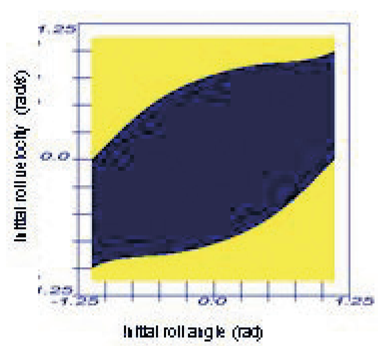
The form of the safe basin in the initial values plane for the ship with and without the bilge keels is similar, but the safe basin for the ship without the bilge keels shows pronounced dependence of capsizing on the period of exposure to the excitation. Such illustrations in the excitation plane or initial value plane can serve as orientational values of safe and unsafe area for ships, and at the same time their fractal nature shows that a small change of the external condition can cause the transition of ship from rolling to capsizing area.

7.4 Calculation of rolling and capsizing in sea waves

Beck [26] and conclusions of the 23rd ITTC Conference [27] point out the great importance of the development of such analysis.

In Figures 17, 18 and 19 the safe basins in the plane of initial conditions without external excitation are shown. The basin area for the ship without the bilge keels is 75% of the basin area for

Figure 17 **Safe basin in the initial value plane, intact ship**
Slika 17 **Bazen sigurnosti u ravnini početnih uvjeta, neoštećeni brod**



the ship with the bilge keels, which indicates the great influence of the bilge keels on the stability against capsizing.

The basin area for the damaged ship is 71.2% of the basin area of the intact ship without the bilge keels.

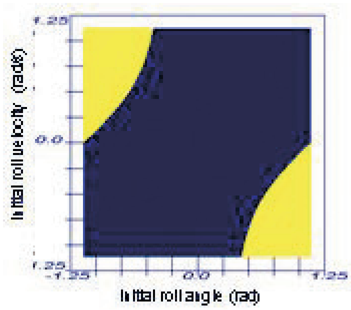


Figure 18 **Safe basin in the initial value plane, intact ship with bilge keels**
Slika 18 **Bazen sigurnosti u ravnini početnih uvjeta, neoštećeni brod s ljuljnim kobilicama**

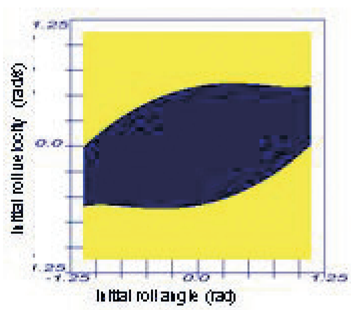


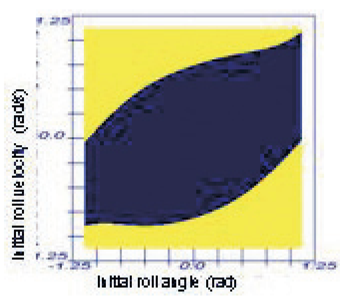
Figure 19 **Safe basin in the initial value plane, damaged ship**
Slika 19 **Bazen sigurnosti u ravnini početnih uvjeta, oštećeni brod**

The ship's behaviour at the heading angles χ=0-180°, significant wave heights $H_{1/3}$=3.75 m and 5 m and speeds V=2, 3 and 4 m/s is observed.

For the intact ship with the bilge keels there is no change in the safe basin area at any of the observed conditions.

For the intact ship without the bilge keels at the speed V=2 m/s, there is no basin erosion greater than 0.3%. For the angles $\chi=\frac{\pi}{2}-\pi$ there is no significant basin erosion for the investigated speeds and wave heights. At the speed V=3 m/s there is no observable erosion at the wave height $H_{1/3}$=3.75 m, but at the wave height $H_{1/3}$=5 m mild basin erosion occurs in-between 5π/18-7π/18. At the speed V=4 m/s the development of basin erosion is similar at the both investigated wave heights, only the values are somewhat larger for the wave height $H_{1/3}$=5 m, see Figures 20 and 21. The influence of the speed increment is greater than the influence of the significant wave height increment.

Figure 20 **Safe basin in the initial value plane, intact ship, area reduced 0.31%**
Slika 20 **Bazen sigurnosti u ravnini početnih uvjeta, neoštećeni brod, površina smanjena 0,31%**
V=3 m/s, $H_{1/3}$=5 m, χ=70°



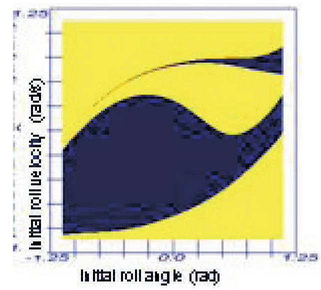


Figure 21 **Safe basin in the initial value plane, intact ship, area reduced 27.5%**
Slika 21 **Bazen sigurnosti u ravnini početnih uvjeta, neoštećeni brod, površina smanjena 27,5%**
V=4 m/s, $H_{1/3}$=3.75 m, χ=70°,

The damaged ship shows also almost no sensitivity for the heading angles $\chi=\frac{\pi}{2}-\pi$ for all investigated speeds and wave heights. At the angles $\chi=0-\frac{\pi}{2}$ basin erosion occurs. At the wave height $H_{1/3}$=3.75 m and speeds V=2 and 3 m/s there is no

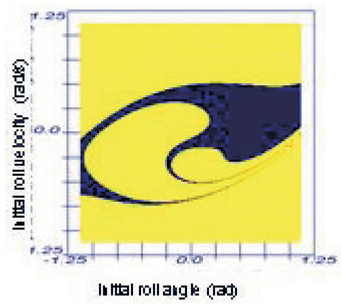
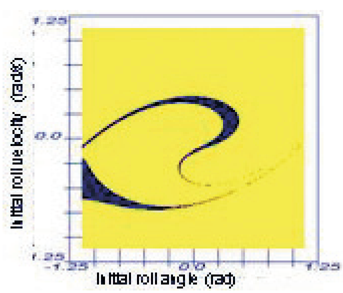


Figure 22 **Safe basin in the initial value plane, damaged ship, area reduced 54.4%**
Slika 22 **Bazen sigurnosti u ravnini početnih uvjeta, oštećeni brod, površina smanjena 54,4%**
V=4 m/s, $H_{1/3}$=3.75 m, χ=55°

Figure 23 **Safe basin in the initial value plane, damaged ship area reduced 86.8%**
Slika 23 **Bazen sigurnosti u ravnini početnih uvjeta, oštećeni brod, površina smanjena 86,8%**
V=4 m/s, $H_{1/3}$=5 m, χ=55°



significant basin erosion. At $H_{1/3}$=5.00 m and V=3 m/s the erosion is significant for the heading angles range 2.5π/18-5π/18.

At the speed V=4 m/s erosion is mild in the heading angle range χ=6π/18-9π/18, but the safe basin almost disappears for the heading angles 3π/18-6π/18, see Figures 22 and 23.

If the damaged ship with the presumption of only linear damping is considered in the specified conditions, the safe basins have a highly fractal structure, and their area is much smaller compared to the area of the safe basins when the cubic damping coefficient is included in the equation. This implies the great importance of taking into account the nonlinearity of damping. The development of basin erosion in this case is shown in Figures 24-27.

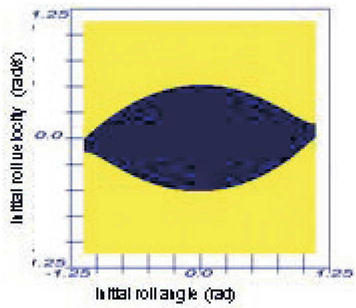


Figure 24 **Safe basin in the initial value plane for damaged ship, without external excitation, assumption of linear damping**
Slika 24 **Bazen sigurnosti u ravnini početnih uvjeta za oštećeni brod, bez vanjske uzbude, pretpostavka linearnog prigušenja**

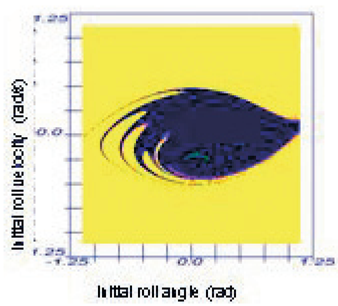
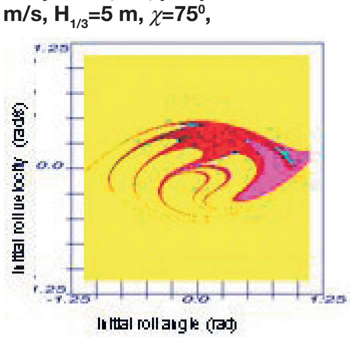


Figure 25 **Safe basin in the initial value plane, basin area reduced 28.2%, assumption of linear damping**
Slika 25 **Bazen sigurnosti u ravnini početnih uvjeta, površina bazena smanjena 28,2%, pretpostavka linearnog prigušenja**
V=2 m/s, $H_{1/3}$=3.75 m, χ=55^0,

Figure 26 **Safe basin in the initial value plane, basin area is 97.1% reduced, assumption of linear damping**
Slika 26 **Bazen sigurnosti u ravnini početnih uvjeta, površina bazena smanjena 97,1%, pretpostavka linearnog prigušenja**
V=4 m/s, $H_{1/3}$=5 m, χ=75^0,



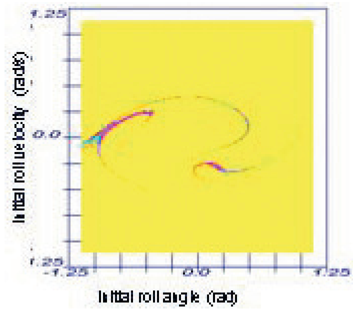


Figure 27 **Safe basin in the initial value plane, assumption of linear damping**
Slika 27 **Bazen sigurnosti u ravnini početnih uvjeta, pretpostavka linearnog prigušenja**
V=4 m/s, $H_{1/3}$=5 m, χ=55^0

7.5 Ship survavibility probabilistic diagram

The ratio of the areas of the basins of erosion with and without wave excitation can be used as the measure of ship survival probability estimation. Construction of such diagrams gives the probability of capsizing as the function of the following three parameters: ship speed, heading angle, significant wave height. The navigation conditions in which there is imminent danger of ship capsizing are defined by these parameters. Figures 28 and 29 represent such diagrams for the intact and damaged ship respectively.

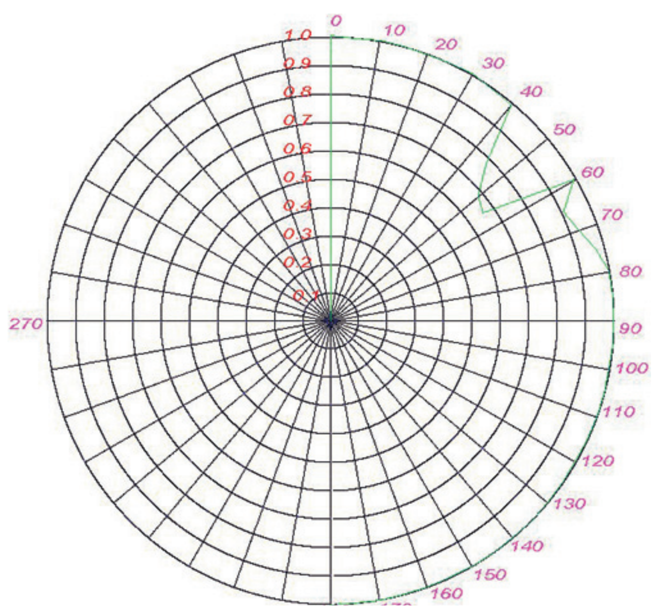


Figure 28 **Ship survivability diagram for the intact ship**
Slika 28 **Dijagram preživljavanja broda, neoštećeni brod**
V=4 m/s, $H_{1/3}$=5.00 m

This diagram represents the unification of the numerical method of harmonic acceleration and the knowledge on nonlinear dynamics.

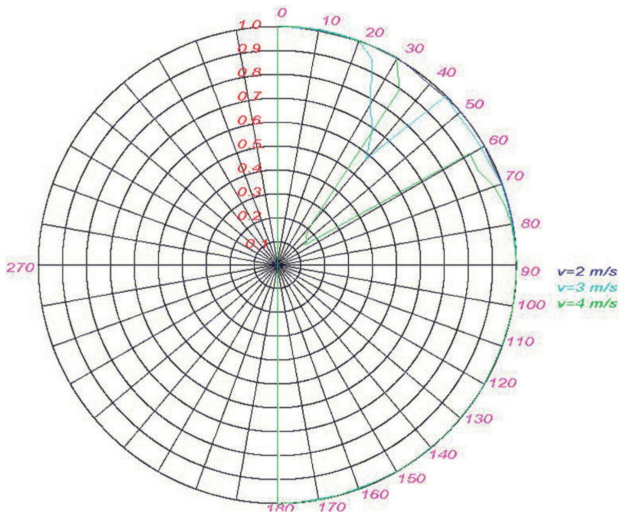


Figure 29 **Ship survivability probabilistic diagram for the damaged ship**

Slika 29 **Dijagram preživljavanja broda, oštećeni brod $H_{1/3}$=5.00 m**

8 Conclusion

In order to raise the level of ship safety it is necessary to work systematically on improving ship design methods, guidance for ship handling and rules and recommendations concerning the ship safety.

The numerical method (harmonic acceleration method) in time domain for solving the nonlinear equation of rolling is presented in this paper. Based on the obtained results, the evaluation of the probability of capsizing for specific ship conditions is done using the method of basin erosion. The ship survivability diagram is presented as the final result (probability of capsizing depending on ship speed, significant wave height and the heading angle).

Such diagram represents:

- contribution to the stability criteria connected with the function of the probability of ship survival,
- contribution to the definition of non-permissible conditions for ship's voyage.

Up to date achievements in this area point to the importance of taking into account nonlinearity in the problem and also the importance of the probabilistic approach.

The suggested procedure should be improved in several ways:

- rolling should be coupled with other degrees of freedom of ship motion
- quantitative definition of ship characteristics in the damaged condition
- variation of restoring moment (function of heading angle)
- possibilities of direct determination of probability (reduction of great number of numerical simulations required).

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