

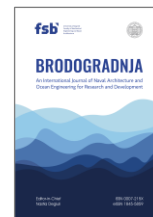


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Adaptive track control for unmanned surface vessels using model predictive control: accounting for water depth and speed variations



Hongwei He^{1,2*}, Luca Donatini¹, Jeroen Verwilligen³, Guillaume Delefortrie¹

¹Ghent University, Ghent, Belgium

²Hunan University, China

³Flanders Hydraulics, Antwerp, Belgium

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ABSTRACT

Model Predictive Control (MPC) is applied to track control of unmanned surface vessels. To adapt to dynamic changes in speed and water depth, a water depth and speed dependent predictive model, with coefficients identified from free running tests (e.g., acceleration and zigzag trials), is developed. The nonlinear least square method is used for parameter estimation of the model. Furthermore, a novel self-tuning method adapts MPC's essential parameter, prediction horizon, to diverse operating conditions. Validation via manoeuvring simulations of the full-scale benchmark KVLCC2 ship covers under keel clearances (UKC) from 20 to 100 % ship's draft (from shallow water to medium deep water) and vessel speeds of 7-15 knots, spanning 25 distinct scenarios. The designed controller achieves high-accuracy path following across all conditions, demonstrating exceptional adaptability to parameter variations.

1. Introduction

Autonomous ships are a significant research topic, especially with the rapid advancements in artificial intelligence. In the realm of ship manoeuvring, this involves motion control, where developing manoeuvring models and designing track controllers are primary research tasks. Navigating in coastal or inland waters poses unique challenges to large surface vessels. The confined space induces changes in the fluid pressure field around the hull, consequently generating hydrodynamic disturbances. Well-known hydrodynamic phenomena include shallow water effects, bank effects and ship-ship interactions [1]. These effects increase the ship's resistance and sinkage, cause speed loss, and can jeopardize the manoeuvrability and controllability of ships, making motion control more challenging. Additionally, in order to increase the shipping efficiency, ship sizes are gradually growing up, leading to more frequent and more extreme confined water effects. Therefore, developing track control strategies that can adapt to these confined water effects is significant to autonomous shipping.

This research highlights the adaptability of the control system to changes in water depth. Crane [2] conducted a series of full-scale sea trials with the 278,000-ton crude oil carrier Esso Osaka in both shallow

* Corresponding author.

E-mail address: hongweihe@hnu.edu.cn

and deep waters at Under Keel Clearance (UKC) ratios to draft of 20, 50 and 320 %. It was found out that the turning circle radius generally increases with decreasing UKC, and overshoot angles in zigzag tests are much smaller in shallow water, revealing the shallow water effects in the ship's course-checking ability. The effects on manoeuvrability have been investigated throughout several decades [3-9].

Apart from the water depth effect, the control system proposed in this research also deals with the ship speed effects, which is a common challenge faced by both inland and seagoing ships. Chen et al. [10] confirmed with free running tests that the Nomoto model parameters significantly vary with ship speed, indicating noticeable changes in vessels' dynamics. However, most researchers only design and validate their control systems at a single speed, and therefore the effectiveness of these systems cannot be guaranteed as speed varies. In this paper, all evaluations were performed at a propeller rate corresponding to self-propulsion condition at the evaluated UKC.

Fossen [11] suggested to determine the parameters of the PID controller for heading/ course keeping based on Nomoto model parameters. This means that the design parameters of a controller should be adjusted when vessels' dynamics changes, otherwise the performance of the controller will be significantly influenced. Adaptive control techniques should be applied to deal with this aspect. For model-free control methods, adaptive control focuses on tuning the design parameters in real time according to the system's dynamic response. A common trend is to implement intelligent optimization algorithms to modify PID controller's parameters in order to be adaptive to various operating conditions of the controlled ship [12-14]. For model-based control methods, adaptive control is usually achieved by adapting the reference model (or control model) utilized in the design process [15] or adjusting the control laws according to the difference between the reference model's output and the real system's response [16, 17].

Chen et al. [10] developed an adaptive PID controller for ship path following at various water depths and speeds, where the optimal PID parameters are formulated as functions of the water depth and speed. The adaptive controller delivered satisfactory performance in extensive experimental tests of polyline path following. However, PID control is prone to destabilization under complex disturbances, and it is difficult to achieve multi-objective co-optimization, such as an integration of track control and collision avoidance considering navigation safety and regulations. Model Predictive Control (MPC) has been applied to curved-path following using the Serret-Frenet frame in the authors' previous work [18]. That controller employed a simple Nomoto-model-based predictor and was experimentally validated around a nominal operating speed. More recently, the controller was extended to speed-adaptive path following and trajectory tracking at a fixed shallow-water condition [19]. Simplified speed-dependent 3-DOF predictive models were developed, while the prediction horizon was adjusted using speed-related scaling laws based on ship speed, propeller rate, or their combination. Nevertheless, the water depth in [19] was fixed at 25 % UKC. Consequently, neither the predictive model nor the horizon-tuning schemes explicitly accounted for variations in manoeuvring dynamics caused by changes in water depth. The present study extends this previous speed-adaptive MPC approach to simultaneous variations in water depth and vessel speed. This extension is achieved through a Norrbinn-based 3-DOF predictive model parameterized by UKC and speed, together with a dynamics-informed relationship between the optimal nondimensional prediction horizon and the combined Nomoto-based manoeuvring index T'_K . The contribution therefore lies in generalizing the previous speed-only adaptation at a fixed water depth to a two-dimensional water-depth–speed operating envelope.

Relative to the authors' previous speed-adaptive MPC study conducted at a fixed water depth [19], the specific contributions of the present work are summarized as follows:

- Water depth and speed dependent predictive model: a parameterized 3-DOF manoeuvring model based on the Norrbinn formulation is developed to incorporate the effects of UKC and vessel speed. The present model provides a unified representation over a two-dimensional water-depth–speed operating envelope, and its coefficients are systematically identified from high-fidelity free-running simulation data collected at selected operating conditions.
- Adaptive prediction horizon to vessel dynamics: a novel adaptive tuning logic for the MPC prediction horizon is introduced. An empirical relationship is established between the optimal nondimensional prediction horizon and the combined Nomoto-based manoeuvring index T'_K , thereby ensuring optimal

tracking accuracy and preventing system instability. Because T'_K reflects changes in vessel response induced jointly by UKC and speed, the proposed relationship generalizes the previous speed-only scaling rule to combined water-depth and speed variations.

- Comprehensive validation: the proposed adaptive framework is evaluated on the full-scale KVLCC2 benchmark vessel over 25 combinations of UKC and speed, including water-depth conditions not directly used for model identification. This extends the validation domain from the fixed water depth experiments in [19] to combined variations in water depth and vessel speed. The high performance and practical engineering value of the proposed method are quantitatively demonstrated.

The remainder of this paper is organized as follows: Section 2 introduces the mathematical modelling of the water depth and speed dependent ship manoeuvring dynamics and the system identification procedure. Section 3 presents the design of the adaptive MPC, detailing the self-tuning logic for the prediction horizon. It also provides a comprehensive discussion of the simulation results, analysing the controller's adaptability. Finally, Section 4 draws the conclusions and outlines directions for future research. For the sake of completeness and reproducibility, the explicit mathematical formulation of the MPC framework is detailed in Appendix A.

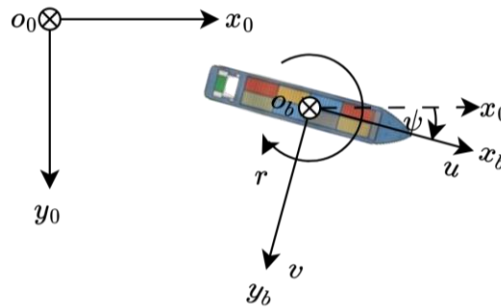


Fig. 1 Earth-fixed frame and body-fixed frame

2. Manoeuvring models

2.1 Overview

State variables of ship manoeuvring motion can be defined in two coordinate systems: the earth-fixed frame and the body-fixed frame, as illustrated in Figure 1. The earth-fixed frame $o_0-x_0y_0z_0$ is static, with its origin o_0 defined at any point on the Earth. The $o_0-x_0y_0$ plane is located on the water surface, and the vertical z_0 axis points downward. The body-fixed frame is attached to the hull: the x_b axis is aligned with the mid-longitudinal section of the hull and points to the bow; the y_b axis points to the starboard side; the $o_b-x_by_b$ plane is parallel to the water surface; the z_b axis is also on the mid-longitudinal section, pointing towards the ship's bottom.

For manoeuvring surface ships, the interest typically lies in horizontal plane motions, and the degrees of freedom (DOF) are then reduced to three: surge, sway and yaw. Considering the hull is usually symmetrical in the transverse direction (y -coordinate of the centre of gravity $y_g = 0$), a 3-DOF manoeuvring model in the body-fixed frame of reference can be formulated as:

$$\begin{aligned} X &= m(\dot{u} - vr - x_g r^2) \\ Y &= m(\dot{v} + ur + x_g \dot{r}) \\ N &= I_z \dot{r} + mx_g(\dot{v} + ur) \end{aligned} \quad (1)$$

where X , Y and N are the hydrodynamic forces and moment respectively for surge, sway and yaw, x_g is the x -coordinate of the centre of gravity, u , v and r are respectively the surge velocity, sway velocity and yaw rate, m is the mass, and I_z is the moment of inertia relative to the z_b axis. The velocity-dependent terms on the

right-hand side of the equation are fictitious forces that arise due to the fact that the equations of motion are solved in the body-fixed reference frame, which is non-inertial.

The coordinates and heading of the ship in the earth-fixed frame are determined by a coordinate system rotation:

$$\begin{aligned}\dot{\boldsymbol{\eta}} &= \mathbf{R}(\psi)\mathbf{v} \\ \boldsymbol{\eta} &= [x; y; \psi] \\ \mathbf{v} &= [u; v; r] \\ \mathbf{R}(\psi) &= \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix}\end{aligned}\quad (2)$$

where $\boldsymbol{\eta}$ denotes the position and orientation vector in the earth-fixed frame, $\mathbf{v}$ denotes the velocity vector in the body-fixed frame, $\dot{\boldsymbol{\eta}}$ denotes the velocity vector in the earth-fixed frame and $\mathbf{R}(\psi)$ is the rotation matrix in the horizontal plane.

Significant effort has been invested in developing expressions of the hydrodynamic forces and moments, which is the crucial distinction between the two main groups of manoeuvring models: whole-ship models and modular models. The whole-ship models expand the hydrodynamic forces and moments at an operating point of the ship using Taylor series. In contrast, the modular models separately establish expressions for the forces and moments acting on the hull, propeller(s) and rudder(s) and then combine them considering the interactions between each two. Table 1 summarizes the classification of manoeuvring models, where δ and n are the rudder angle and propeller rate, subscripts H, R and P respectively denote the hull, rudder and propeller.

The modular models are widely used in ship manoeuvring simulators since they offer more flexibility and are more applicable in complex operating conditions (such as four quadrants of propeller). However, whole-ship models have a simple form and can be converted to polynomial functions that facilitate the parameter estimation, and thus they are more popular in parametric modelling and motion control.

Table 1 Classification of manoeuvring models

Model type	General expression
Whole-ship model	$\begin{cases} X = X(\dot{u}, \dot{v}, \dot{r}, u, v, r, \delta, n) \\ Y = Y(\dot{u}, \dot{v}, \dot{r}, u, v, r, \delta, n) \\ N = N(\dot{u}, \dot{v}, \dot{r}, u, v, r, \delta, n) \end{cases}$
Modular model	$\begin{cases} X = X_H + X_R + X_P \\ Y = Y_H + Y_R + Y_P \\ N = N_H + N_R + N_P \end{cases}$

2.2 Norrbbin model

Norrbbin [20] employed the bis-system normalization method to derive a 3-DOF nonlinear manoeuvring model for large ships in deep and confined water. The bis-system uses the time unit $\sqrt{L/g}$ (where L is the ship length, and g is the gravitational acceleration) as the normalization variable, allowing for the inclusion of zero-speed and high-speed applications. The general form of the Norrbbin model is:

$$\begin{aligned}\dot{u} - vr - Lx_g''r^2 &= gX'' \\ \dot{v} + ur + Lx_g''\dot{r} &= gY'' \\ (Lk_z'')^2\dot{r} + Lx_g''(\dot{v} + ur) &= gLN''\end{aligned}\quad (3)$$

The dimensionless quantities, denoted by "", are defined using the ship length L , the hull volume ∇ , and the water density ρ as follows: $k_z'' = L^{-1}\sqrt{I_z/m}$, $x_g'' = L^{-1}x_g$, $X'' = \frac{X}{\rho g \nabla}$, $Y'' = \frac{Y}{\rho g \nabla}$ and $N'' = \frac{N}{\rho g \nabla L}$, where X , Y and N are hydrodynamic forces and moment expressed as nonlinear functions.

Norrbin extended the whole-ship model in deep water to the confined water case, based upon theoretical results, special calculations and experiments. The forces and moments in deep and confined water were considered as the linear superposition of those in deep open water and the hydrodynamic interferences caused by confinements in the waterway, e.g., the presence of port, starboard side wall restrictions and bottom depth limitations. These interferences can be categorized into two groups, shallow water effects and bank effects.

The shallow water effects mainly increase resistance and sinkage and decrease manoeuvrability. The Norrbin model adopted a water depth parameter to form additional terms that describe the hydrodynamic interferences induced by shallow water:

$$\zeta = \frac{1}{UKC} = \frac{T_m}{h-T_m} \quad (4)$$

where T_m is the ship draft, and h is the water depth.

The bank effect is generated in an asymmetric flow field induced by the lateral confinements of the water. Lateral forces act on the ship body, commonly known as bank suction and ship-bank interaction. Bank distance parameters (port and starboard) are utilized to describe the bank effect. The asymmetric forces acting on the ship highly increase with ζ , and the general model includes complex coupling. However, this study only focuses on introducing the shallow water effect, hence terms related to bank effects terms will be decoupled and removed.

The model in [20] also employs hydrodynamic derivatives. The nonlinear hydrodynamic functions are expressed in the form:

$$\begin{aligned} X'' &= X''(\dot{u}, u, v, r, c, T_p, \delta, \zeta) \\ Y'' &= Y''(\dot{v}, u, v, r, c, T_p, \delta, \zeta) \\ N'' &= N''(\dot{r}, u, v, r, c, T_p, \delta, \zeta) \end{aligned} \quad (5)$$

where c is the flow velocity at the rudder, and T_p is the propeller thrust. c and T_p are functions of u and n . Corresponding hydrodynamic derivatives are then defined. Specifically, the Norrbin model considering the shallow water effect is formed as [21-23]:

$$\begin{aligned} &L(1 - X''_{\dot{u}} - X''_{\dot{u}\zeta})\dot{u} \\ &\quad = X''_{uu}u^2 + L(1 + X''_{vr})vr + X''_{vv}v^2 + X''_{c|c|\delta\delta}c|c|\delta^2 \\ &\quad + X''_{c|c|\beta\delta}c|c|\beta\delta + LgT_p''(1 - t_p) + X''_{uu\zeta}u^2\zeta + LX''_{vr\zeta}vr\zeta \\ &\quad + X''_{vv\zeta\zeta}v^2\zeta^2 \\ &L(1 - Y''_{\dot{v}} - Y''_{\dot{v}\zeta})\dot{v} \\ &\quad = Y''_{uv}uv + Y''_{|v|v}|v|v + Y''_{c|c|\delta}c|c|\delta + L(Y''_{ur} - 1)ur \\ &\quad + Y''_{c|c|\beta|\delta}c|c|\beta|\delta + LY''_TgT_p'' + LY''_{ur\zeta}ur\zeta + Y''_{uv\zeta}uv\zeta \\ &\quad + Y''_{v|v|\zeta}v|v|\zeta + Y''_{c|c|\beta|\delta\zeta}c|c|\beta|\delta\zeta \\ &L^2(K''_{zz} - N''_{\dot{r}} - N''_{\dot{r}\zeta})\dot{r} \\ &\quad = N''_{uv}uv + LN''_{|v|r}|v|r + N''_{c|c|\delta}c|c|\delta + L(N''_{ur} - x''_g)ur \\ &\quad + N''_{c|c|\beta|\delta}c|c|\beta|\delta + LN''_TgT_p'' + LN''_{ur\zeta}ur\zeta + N''_{uv\zeta}uv\zeta \\ &\quad + LN''_{|v|r\zeta}|v|r\zeta + N''_{c|c|\beta|\delta\zeta}c|c|\beta|\delta\zeta \end{aligned} \quad (6)$$

where $\beta = \tan^{-1} \frac{v}{u}$ or $\beta \approx \frac{v}{u}$ is the drift angle, t_p is the thrust deduction factor, and:

$$\begin{aligned} gT_p'' &= L^{-1}T''_{uu}u^2 + T''_{un}un + LT''_{|n|n}|n|n \\ c^2 &= c_{un}un + c_{nn}n^2 \end{aligned} \quad (7)$$

where the constants for determining c^2 depend on the screw characteristics and the wake factor.

2.3 Tabular model

Flanders Hydraulics proposed a tabular manoeuvring model in shallow and confined water, which is a type of modular model [24]. The model is applicable across a wide range of UKC and in four quadrants of

forward speed and propeller rate combinations. For each UKC, a modular model is developed separately based on captive tests. The forces and moment acting on the hull are expressed as:

$$\begin{aligned}
 X_H &= X_u \dot{u} + \frac{1}{2} \rho L T \left\{ (u^2 + v^2) [X'(\beta)] + \left[u^2 + \left(\frac{1}{2} r L \right)^2 \right] X'(\gamma) \right. \\
 &\quad \left. + \left[v^2 + \left(\frac{1}{2} r L \right)^2 \right] X'(\chi) \right\} \\
 Y_H &= Y_v \dot{v} + Y_r(\beta) \dot{r} \\
 &\quad + \frac{1}{2} \rho L T \left\{ (u^2 + v^2) [Y'(\beta)] + \left[u^2 + \left(\frac{1}{2} r L \right)^2 \right] Y'(\gamma) \right. \\
 &\quad \left. + \left[v^2 + \left(\frac{1}{2} r L \right)^2 \right] Y'(\chi) \right\} \\
 N_H &= N_v \dot{v} + N_r(\beta) \dot{r} \\
 &\quad + \frac{1}{2} \rho L^2 T \left\{ (u^2 + v^2) Y'(\beta) x_Y(\beta) + \left[u^2 + \left(\frac{1}{2} r L \right)^2 \right] N'(\gamma) \right. \\
 &\quad \left. + \left[v^2 + \left(\frac{1}{2} r L \right)^2 \right] N'(\chi) \right\}
 \end{aligned} \tag{8}$$

The hydrodynamic derivatives and functions in the model are tabulated with respect to the drift angle β , the yaw rate angle γ and the drift-yaw correlation angle χ , which are defined as:

$$\begin{aligned}
 \beta &= \arctan \left(-\frac{v}{u} \right) \\
 \gamma &= \arctan \left(\frac{r L}{u 2} \right) \\
 \chi &= \arctan \left(\frac{r L}{v 2} \right)
 \end{aligned} \tag{9}$$

Most hydrodynamic derivatives and functions are derived for a discrete set of under keel clearance parameters ζ ; for each ζ , a different set of manoeuvring coefficients, making up a complete manoeuvring model, is available. For an arbitrary ζ , the hull forces are calculated for the two closest models and are then linearly interpolated based on the actual value of ζ . Only one model is considered when ζ is outside of the range for which models are defined, thus no extrapolations are performed with respect to ζ . The modelling of the rudder and propeller forces is similar to those in the MMG model, but four quadrants are included in the FH propeller model [24].

2.4 Least square method

The basic idea of the least square method is to estimate a set of parameters that make the model output as close to the real system output as possible. The degree of closeness is measured by the sum of the squared errors (SE) between the model output and the real system output. A linear system whose output is a linear combination of the input can be expressed as:

$$\mathbf{y} = \mathbf{x} \boldsymbol{\theta} \tag{10}$$

where $\mathbf{x}$ is the input vector, $\mathbf{y}$ is the output vector, and $\boldsymbol{\theta}$ is the parameter vector. To estimate the parameters, M input-output pairs are sampled. The SE becomes:

$$SE = \sum_{k=1}^M (\mathbf{y}_k - \mathbf{x}_k \hat{\boldsymbol{\theta}})^2 \tag{11}$$

where $\hat{\boldsymbol{\theta}}$ is an estimated parameter vector. This equation can be rewritten in the matrix form:

$$\begin{aligned}
 SE &= (\mathbf{Y} - \mathbf{X} \hat{\boldsymbol{\theta}})^T (\mathbf{Y} - \mathbf{X} \hat{\boldsymbol{\theta}}) \\
 \mathbf{Y} &= [\mathbf{y}_1; \mathbf{y}_2; \dots; \mathbf{y}_M] \\
 \mathbf{X} &= [\mathbf{x}_1; \mathbf{x}_2; \dots; \mathbf{x}_M]
 \end{aligned} \tag{12}$$

SE reaches its minimum value while $\nabla SE = 0$, i.e.,

$$-2\mathbf{X}^T\mathbf{Y} + 2\mathbf{X}^T\mathbf{X}\hat{\boldsymbol{\theta}} = 0 \quad (13)$$

Therefore, the optimal estimation of the parameter vector is:

$$\hat{\boldsymbol{\theta}}^* = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y} \quad (14)$$

This solution is called the least square solution.

For a nonlinear system, described as:

$$\mathbf{y} = f(\mathbf{x}, \boldsymbol{\theta}) \quad (15)$$

the least square method can also be applied to estimate the parameter vector, but it requires an iterative optimization. The nonlinear system is linearized using the Taylor series expansion technique at each iteration point as:

$$\hat{\mathbf{y}} = f(\mathbf{x}, \hat{\boldsymbol{\theta}}_i) + \nabla f(\mathbf{x}, \hat{\boldsymbol{\theta}}_i)^T (\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_i) \quad (16)$$

where $\hat{\boldsymbol{\theta}}_i$ is the estimated parameter vector at the i -th iteration step. The SE at the i -th iteration step is then calculated as:

$$SE = \sum_{k=1}^M \left(\mathbf{y}_k - f(\mathbf{x}_k, \hat{\boldsymbol{\theta}}_i) - \nabla f(\hat{\boldsymbol{\theta}}_i, \mathbf{x}_k)^T (\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_i) \right)^2 = (\mathbf{E} - \mathbf{J}(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_i))^T (\mathbf{E} - \mathbf{J}(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_i)) \quad (17)$$

$$\mathbf{E} = [\mathbf{e}_1; \mathbf{e}_2, \dots, \mathbf{e}_M], \quad \mathbf{e}_k = \mathbf{y}_k - f(\mathbf{x}_k, \hat{\boldsymbol{\theta}}_i)^T$$

$$\mathbf{J} = [\nabla f(\hat{\boldsymbol{\theta}}_i, \mathbf{x}_1)^T; \nabla f(\hat{\boldsymbol{\theta}}_i, \mathbf{x}_2)^T; \dots; \nabla f(\hat{\boldsymbol{\theta}}_i, \mathbf{x}_M)^T]$$

Hence, the least square solution is:

$$\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_i = (\mathbf{J}^T\mathbf{J})^{-1}\mathbf{J}^T\mathbf{E} \quad (18)$$

and the update of the estimated parameter vector becomes:

$$\hat{\boldsymbol{\theta}}_{i+1} = \hat{\boldsymbol{\theta}}_i + (\mathbf{J}^T\mathbf{J})^{-1}\mathbf{J}^T\mathbf{E} \quad (19)$$

After a certain number of iterations, if the SE is reduced to the required value, the least square solution can be considered as the optimal solution. The process to get the optimal solution for the nonlinear system is termed the nonlinear least square method.

2.5 Water depth and speed dependent model

The Norrbinn model is utilized to identify a water depth and speed dependent model. Only the first quadrant, i.e. $u > 0$ and $n > 0$, is considered in system identification. Therefore, the validity range of the proposed predictive model is strictly assumed to be forward-only, first-quadrant operations. Four-quadrant propeller behaviours (e.g., crash stops or reversing) are not considered, as they fall outside the scope of the current track controller. The surge dynamics in the Norrbinn model can then be rewritten as:

$$\begin{aligned} \dot{u} = & \frac{(X''_{uu} + X''_{uv\zeta}\zeta + (1 - t_P)T''_{uu})}{Lm_1} u^2 + \frac{(1 + X''_{vr} + X''_{vr\zeta}\zeta)}{m_1} vr \\ & + \frac{(X''_{vv} + X''_{vv\zeta}\zeta^2)}{m_1} v^2 + \frac{X''_{c|c|\delta\delta}c_{un}}{Lm_1} un\delta^2 + \frac{X''_{c|c|\delta\delta}c_{nn}}{Lm_1} n^2\delta^2 \\ & + \frac{X''_{c|c|\beta\delta}c_{un}}{Lm_1} un\beta\delta + \frac{X''_{c|c|\beta\delta}c_{nn}}{Lm_1} n^2\beta\delta + \frac{(1 - t_P)T''_{un}}{m_1} un \\ & + \frac{L(1 - t_P)T''_{|n|n}}{m_1} n^2 \end{aligned} \quad (20)$$

where $m_1 = 1 - X''_{\dot{u}} - X''_{\dot{u}\zeta}\zeta$. This equation is further rewritten into the polynomial form for identification:

$$\dot{u} = a_1 u^2 + a_2 vr + a_3 v^2 + a_4 un\delta^2 + a_5 n^2\delta^2 + a_6 un\beta\delta + a_7 n^2\beta\delta + a_8 un + a_9 n^2 \quad (21)$$

where

$$\begin{aligned} a_1 &= (a_{11} + a_{12}\zeta)/(1 + m_{01}\zeta) \\ a_2 &= (a_{21} + a_{22}\zeta)/(1 + m_{01}\zeta) \\ a_3 &= (a_{31} + a_{32}\zeta^2)/(1 + m_{01}\zeta) \\ a_4 &= a_{41}/(1 + m_{01}\zeta) \\ a_5 &= a_{51}/(1 + m_{01}\zeta) \\ a_6 &= a_{61}/(1 + m_{01}\zeta) \\ a_7 &= a_{71}/(1 + m_{01}\zeta) \\ a_8 &= a_{81}/(1 + m_{01}\zeta) \\ a_9 &= a_{91}/(1 + m_{01}\zeta) \end{aligned} \quad (22)$$

At a single water depth, parameters a_i , $i = 1, \dots, 9$, are constants. According to Equations (10), (12) and (14), they can be easily estimated by using the linear least square method with:

$$\begin{aligned} \mathbf{y} &= \dot{u} \\ \mathbf{x} &= [u^2, vr, v^2, un\delta^2, n^2\delta^2, un\beta\delta, n^2\beta\delta, un, n^2] \\ \boldsymbol{\theta} &= [a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9]^T \end{aligned} \quad (23)$$

The same procedure can be implemented for the parameter estimation of the steering equations.

For the sway dynamics:

$$\begin{aligned} \dot{v} = & \frac{(Y''_{uv} + Y''_{uv\zeta}\zeta)}{Lm_2} uv + \frac{(Y''_{|v|v} + Y''_{|v|v|\zeta}\zeta)}{Lm_2} |v|v + \frac{Y''_{c|c|\delta}c_{un}}{Lm_2} un\delta + \frac{Y''_{c|c|\delta}c_{nn}}{Lm_2} n^2\delta \\ & + \frac{(Y''_{ur} - 1 + Y''_{ur\zeta}\zeta)}{m_2} ur + \frac{c_{un}(Y''_{c|c|\beta|\beta|\delta} + Y''_{c|c|\beta|\beta|\delta\zeta}\zeta)}{Lm_2} un\beta|\beta||\delta| \\ & + \frac{c_{nn}(Y''_{c|c|\beta|\beta|\delta} + Y''_{c|c|\beta|\beta|\delta\zeta}\zeta)}{Lm_2} n^2\beta|\beta||\delta| + \frac{Y''_T T''_{uu}}{Lm_2} u^2 + \frac{Y''_T T''_{un}}{m_2} un \\ & + \frac{LY''_T T''_{|n|n}}{m_2} n^2 \\ m_2 &= 1 - Y''_{\dot{v}} - Y''_{\dot{v}\zeta}\zeta \end{aligned} \quad (24)$$

its polynomial function form for identification is:

$$\begin{aligned} \dot{v} = & b_1 uv + b_2 |v|v + b_3 un\delta + b_4 n^2\delta + b_5 ur + b_6 un\beta|\beta||\delta| + b_7 n^2\beta|\beta||\delta| \\ & + b_8 u^2 + b_9 un + b_{10} n^2 \\ b_1 &= (b_{11} + b_{12}\zeta)/(1 + m_{02}\zeta) \\ b_2 &= (b_{21} + b_{22}\zeta)/(1 + m_{02}\zeta) \\ b_3 &= b_{31}/(1 + m_{02}\zeta) \\ b_4 &= b_{41}/(1 + m_{02}\zeta) \end{aligned} \quad (25)$$

$$\begin{aligned}
b_5 &= (b_{51} + b_{52}\zeta)/(1 + m_{02}\zeta) \\
b_6 &= (b_{61} + b_{62}\zeta)/(1 + m_{02}\zeta) \\
b_7 &= (b_{71} + b_{72}\zeta)/(1 + m_{02}\zeta) \\
b_8 &= b_{81}/(1 + m_{02}\zeta) \\
b_9 &= b_{91}/(1 + m_{02}\zeta) \\
b_{10} &= b_{101}/(1 + m_{02}\zeta)
\end{aligned}$$

For the yaw dynamics:

$$\begin{aligned}
\dot{r} &= \frac{(N''_{uv} + N''_{uv}\zeta)}{L^2 m_3} uv + \frac{(N''_{|v|r} + N''_{|v|r}\zeta)}{L m_3} |v|r + \frac{c_{un} N''_{c|c|\delta}}{L^2 m_3} un\delta + \frac{c_{nn} N''_{c|c|\delta}}{L^2 m_3} n^2 \delta \\
&\quad + \frac{(N''_{ur} - x''_g + N''_{ur}\zeta)}{L m_3} ur + \frac{c_{un}(N''_{c|c|\beta|\delta} + N''_{c|c|\beta|\delta}\zeta)}{L^2 m_3} un\beta|\beta||\delta| \\
&\quad + \frac{c_{nn}(N''_{c|c|\beta|\delta} + N''_{c|c|\beta|\delta}\zeta)}{L^2 m_3} n^2 \beta|\beta||\delta| + \frac{N''_T T''_{uu}}{L^2 m_3} u^2 + \frac{N''_T T''_{un}}{L m_3} un \\
&\quad + \frac{N''_T T''_{|n|n}}{m_3} |n|n \\
m_3 &= K''_{zz} - N''_{\dot{r}} - N''_{\dot{r}\zeta}
\end{aligned} \tag{26}$$

its polynomial function form for identification is:

$$\begin{aligned}
\dot{r} &= c_1 uv + c_2 |v|r + c_3 un\delta + c_4 n^2 \delta + c_5 ur + c_6 un\beta|\beta||\delta| + c_7 n^2 \beta|\beta||\delta| \\
&\quad + c_8 u^2 + c_9 un + c_{10} n^2 \\
c_1 &= (c_{11} + c_{12}\zeta)/(1 + m_{03}\zeta) \\
c_2 &= (c_{21} + c_{22}\zeta)/(1 + m_{03}\zeta) \\
c_3 &= c_{31}/(1 + m_{03}\zeta) \\
c_4 &= c_{41}/(1 + m_{03}\zeta) \\
c_5 &= (c_{51} + c_{52}\zeta)/(1 + m_{03}\zeta) \\
c_6 &= (c_{61} + c_{62}\zeta)/(1 + m_{03}\zeta) \\
c_7 &= (c_{71} + c_{72}\zeta)/(1 + m_{03}\zeta) \\
c_8 &= c_{81}/(1 + m_{03}\zeta) \\
c_9 &= c_{91}/(1 + m_{03}\zeta) \\
c_{10} &= c_{101}/(1 + m_{03}\zeta)
\end{aligned} \tag{27}$$

The identification model for ship manoeuvring at one water depth is:

$$\begin{aligned}
\dot{u} &= a_1 u^2 + a_2 vr + a_3 v^2 + a_4 un\delta^2 + a_5 n^2 \delta^2 + a_6 un\beta\delta + a_7 n^2 \beta\delta + a_8 un \\
&\quad + a_9 n^2 \\
\dot{v} &= b_1 uv + b_2 |v|v + b_3 un\delta + b_4 n^2 \delta + b_5 ur + b_6 un\beta|\beta||\delta| + b_7 n^2 \beta|\beta||\delta| \\
&\quad + b_8 u^2 + b_9 un + b_{10} n^2 \\
\dot{r} &= c_1 uv + c_2 |v|r + c_3 un\delta + c_4 n^2 \delta + c_5 ur + c_6 un\beta|\beta||\delta| + c_7 n^2 \beta|\beta||\delta| \\
&\quad + c_8 u^2 + c_9 un + c_{10} n^2
\end{aligned} \tag{28}$$

2.6 Modelling results

A validation of the identification algorithm was done by applying it to identify the Norrbinn model of the KVLCC2 benchmark ship in full scale dimensions. The KVLCC2 ship was designed to provide data for studying flow physics and validating CFD calculations for a modern tanker hull form. Its main dimensions are shown in Table 2, and other characteristics, geometries, loading conditions, speeds and model tests can be referred to on SIMMAN websites [25]. The data on which the Norrbinn model is identified come from simulations performed with the tabular model proposed by Flanders Hydraulics as described above.

Table 2 Main dimensions of KVLCC2 ship in full scale dimensions

Main dimensions	Value	Main dimensions	Value
Length between perpendiculars (m)	320	Length at the designed waterline	325.5
Beam (m)	58	Draft (m)	20.8
Diameter of propeller (m)	9.86	Maximum RPM	100
Surface of rudder (m ²)	273.3	Turn rate (deg/s)	2.32

Simulations were conducted at three water depths, $2T_m$, $1.5T_m$ and $1.25T_m$ (i.e. $\zeta = 1, 2$ and 4), to generate data for system identification (coefficients of the FH tabular mathematical model are only available for 80, 30 and 20 % UKC. For other UKC's, coefficients were interpolated.). The consumed data was collected from:

1. acceleration tests: initial speed at 0, and constant propeller rate at 100 rpm
2. zigzag: $20^\circ/10^\circ$ starting to the port side and $10^\circ/5^\circ$ starting to the starboard side, 15kn (corresponding self-propulsion rpm was used)
3. turning circle: port 20° , 15kn

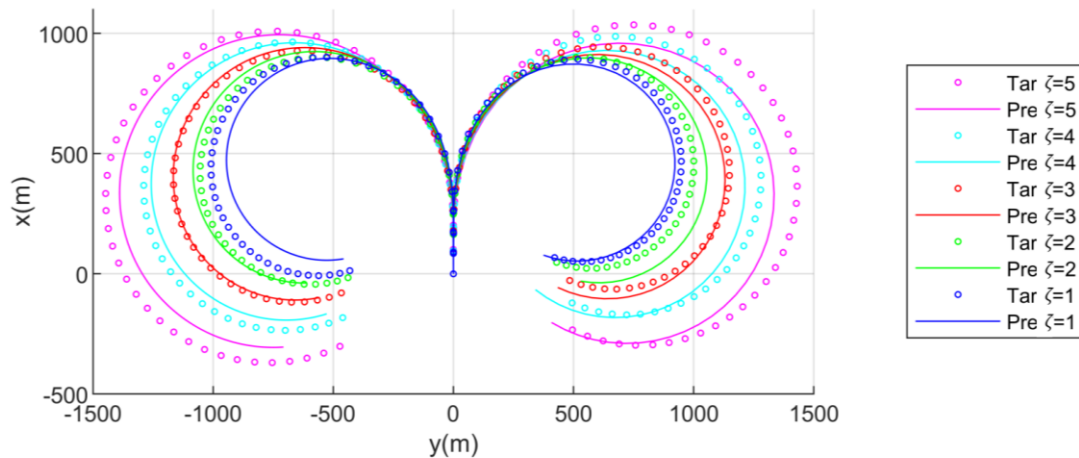
Since manoeuvres performed on both sides (e.g., $20^\circ/10^\circ$ zigzag) exhibit nearly identical responses, data from only one side were selected to streamline the dataset. For distinct manoeuvres, alternating sides were intentionally chosen to enable the identification algorithm to capture inherent asymmetries. While optimizing training data selection could further improve identification accuracy, this approach was not undertaken as system identification is outside the focus of this paper. The nonlinear least square algorithm was implemented for the estimation of Norrbinn model parameters.

2.6.1 Standard manoeuvres at different water depths

Firstly, The Norrbinn model was used to predict manoeuvres at different water depths. It should be noted that the identification dataset was generated at only three water depth conditions, whereas the following validation covers a wider range of water depths. Consequently, the results at water depths not directly included in the identification dataset provide an interpolation/extrapolation evaluation of the identified depth dependent model. The results were compared to the corresponding results of the FH tabular model (referred to as target).

a) Turning circle

Figure 2 illustrates the 35° turning circles conducted at different water depths while operating at 11 knots. It is seen that the predicted trajectories show good agreement with the target trajectories. Table 3 lists the corresponding turning circle indices: advance, transfer, tactical diameter and turning radius. The maximum percentage error of the turning radius is 6.9 %, indicating the high accuracy of the prediction results.

**Fig. 2** 35° Turning circles conducted at different water depths for KVLCC2 at 11 knots

a) Zigzag

Figure 3 shows the time series of the rudder angle and heading for the $10^\circ/10^\circ$ zigzag tests conducted at different water depths. A satisfactory agreement between the predictions and the targets is observed. Table 4 lists the zigzag indices, including initial turning time, first overshoot angle and second overshoot angle. Figure 4 visualizes the overshoot angles. The general trend is well maintained although there is some difference between the predicted and target values. Especially at low UKC ($\zeta = 5$), the Norrbin model presents less asymmetry than the FH tabular model.

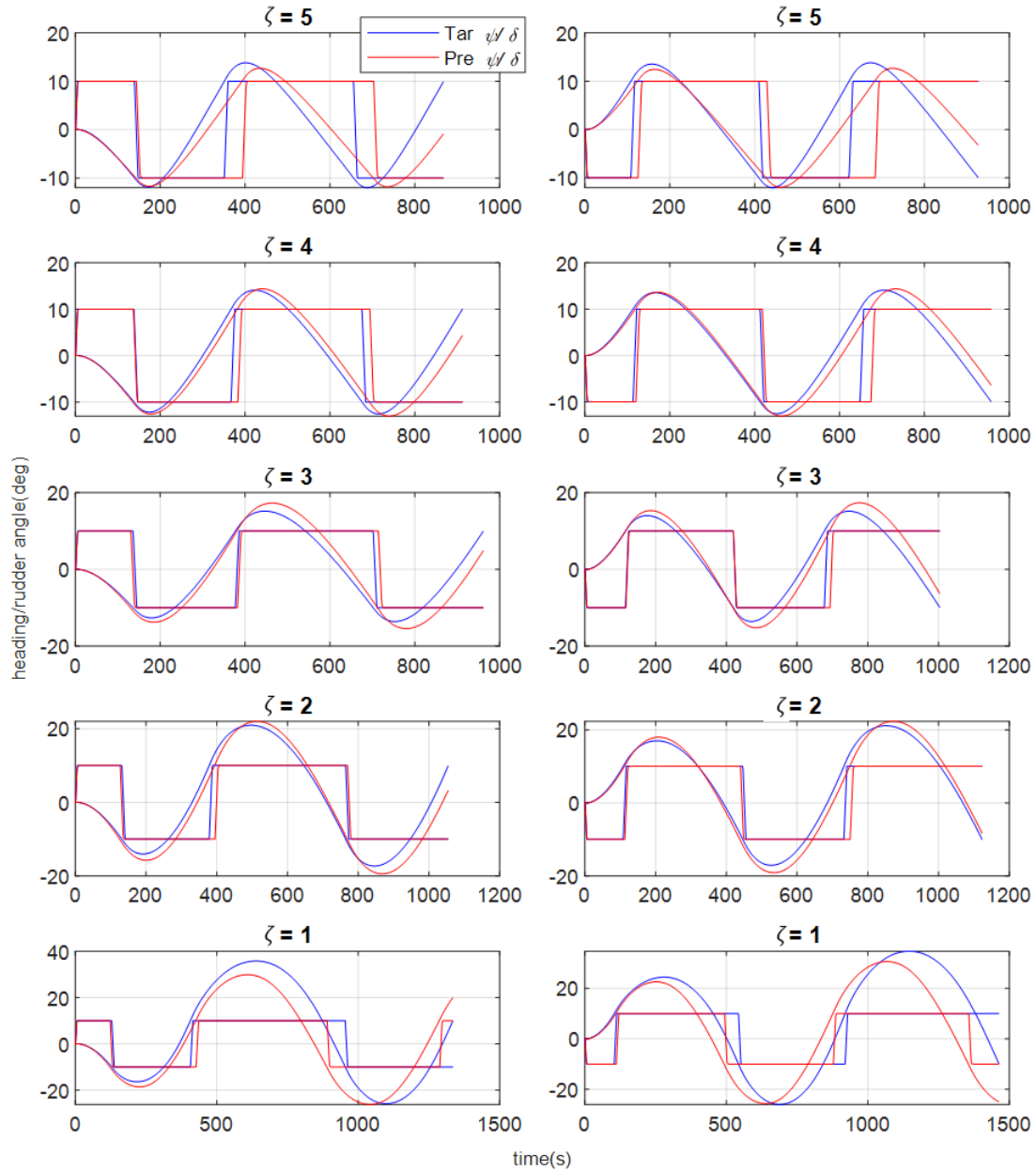


Fig. 3 $10^\circ/10^\circ$ zigzag test conducted at different water depths for KVLCC2 at 11 knots. Left/ right column: zigzags start to the port/ starboard side

Table 3 Turning circle indices for KVLCC2 at 11 knots at different water depths. x_{090} = advance, y_{090} = transfer, y_{0180} = tactical diameter, R_c = turning radius; superscript “’” represents the predicted values

Manoeuvre	Index	$\zeta = 5$	$\zeta = 4$	$\zeta = 3$	$\zeta = 2$	$\zeta = 1$
35° port	R_c [m]	688	598	529	482	460
	R_c' [m]	652	577	531	495	428
	x_{090} [m]	1008	962	925	895	892
	x_{090}' [m]	993	959	937	919	886
	y_{090} [m]	699	621	558	488	442
	y_{090}' [m]	681	609	555	503	418
	y_{0180} [m]	1446	1287	1162	1049	999
	y_{0180}' [m]	1388	1256	1163	1077	934
35° starboard	R_c [m]	663	576	509	449	424
	R_c' [m]	619	550	508	477	415
	x_{090} [m]	1034	985	941	895	882
	x_{090}' [m]	958	927	909	893	862
	y_{090} [m]	697	621	553	467	418
	y_{090}' [m]	650	586	537	484	403
	y_{0180} [m]	1429	1278	1146	995	939
	y_{0180}' [m]	1333	1212	1127	1049	912

Table 4 10°/10° zigzag indices for KVLCC2 at 11 knots at different water depths. t_a = initial turning time, φ_{01} = first overshoot angle, φ_{02} = second overshoot angle, superscript “’” represents the predicted values

Manoeuvre	Index	$\zeta = 5$	$\zeta = 4$	$\zeta = 3$	$\zeta = 2$	$\zeta = 1$
10°/10° port	t_a [s]	140	139	137	132	129
	t_a' [s]	143	136	130	126	122
	φ_{01} [deg]	1.89	2.21	2.68	4.03	6.45
	φ_{01}' [deg]	1.73	2.61	3.78	5.73	8.66
	φ_{02} [deg]	3.81	4.09	5.21	10.94	25.76
	φ_{02}' [deg]	2.68	4.38	7.29	12.01	19.85
10°/10° starboard	t_a [s]	109	114	115	109	104
	t_a' [s]	124	119	115	112	110
	φ_{01} [deg]	3.54	3.55	4.03	7.00	14.46
	φ_{01}' [deg]	2.45	3.64	5.34	7.98	12.58
	φ_{02} [deg]	3.64	2.60	3.59	7.07	16.04
	φ_{02}' [deg]	1.93	3.10	5.25	9.11	15.69

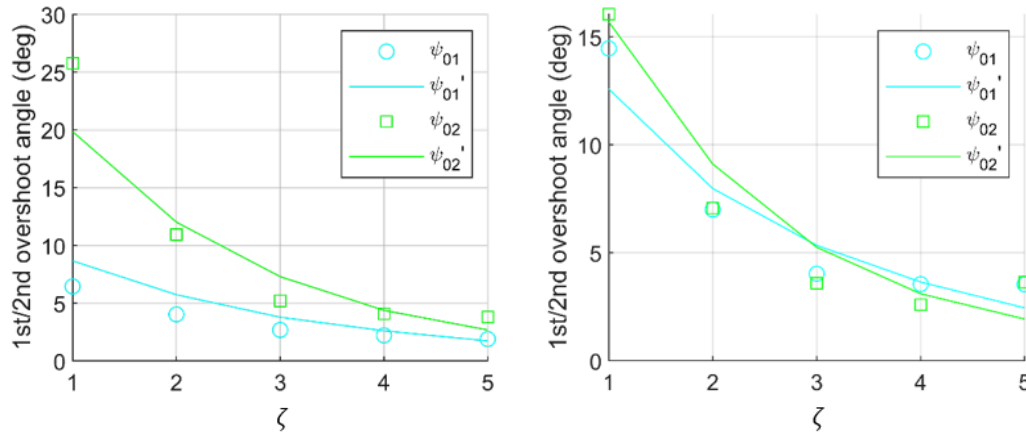


Fig. 4 Overshoot angles of the $10^\circ/10^\circ$ zigzag tests for KVLCC2 at 11 knots at different water depths. Left/ right figure: zigzags start to the port/ starboard side. ψ_{01} = first overshoot angle, ψ_{02} = second overshoot angle, superscript “'” represents the predicted values

2.6.2 Standard manoeuvres at different speeds

Subsequently, manoeuvres were conducted at different speeds to further test the identified model. The water depth was set to $1.25T_m$ ($\zeta = 4$), and the tested speeds included 7, 9, 11, 13 and 15 knots (corresponding propeller rates at self-propulsion points were used for the FH model. The identified model used the same propeller rates as the FH model did.). Figure 5 illustrates the predicted 35° turning circles at different speeds along with their corresponding targets. Figure 6 shows the $10^\circ/10^\circ$ zigzag tests performed at various speeds. The comparative results suggest a good prediction accuracy of the identified model across different speeds, demonstrating the effectiveness of the proposed identification method.

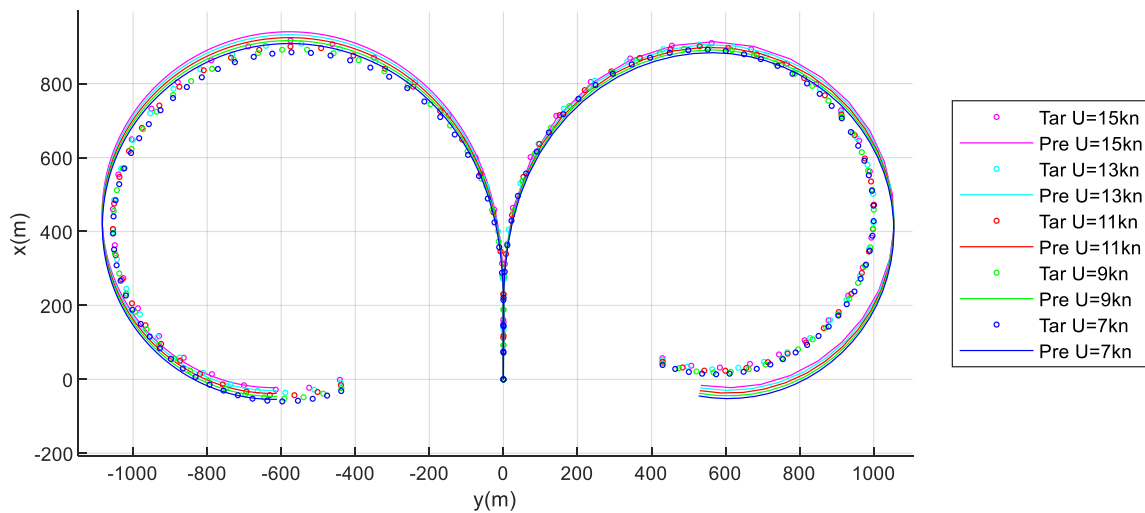


Fig. 5 35° turning circles conducted at different speeds for KVLCC2 at water depth $\zeta = 4$

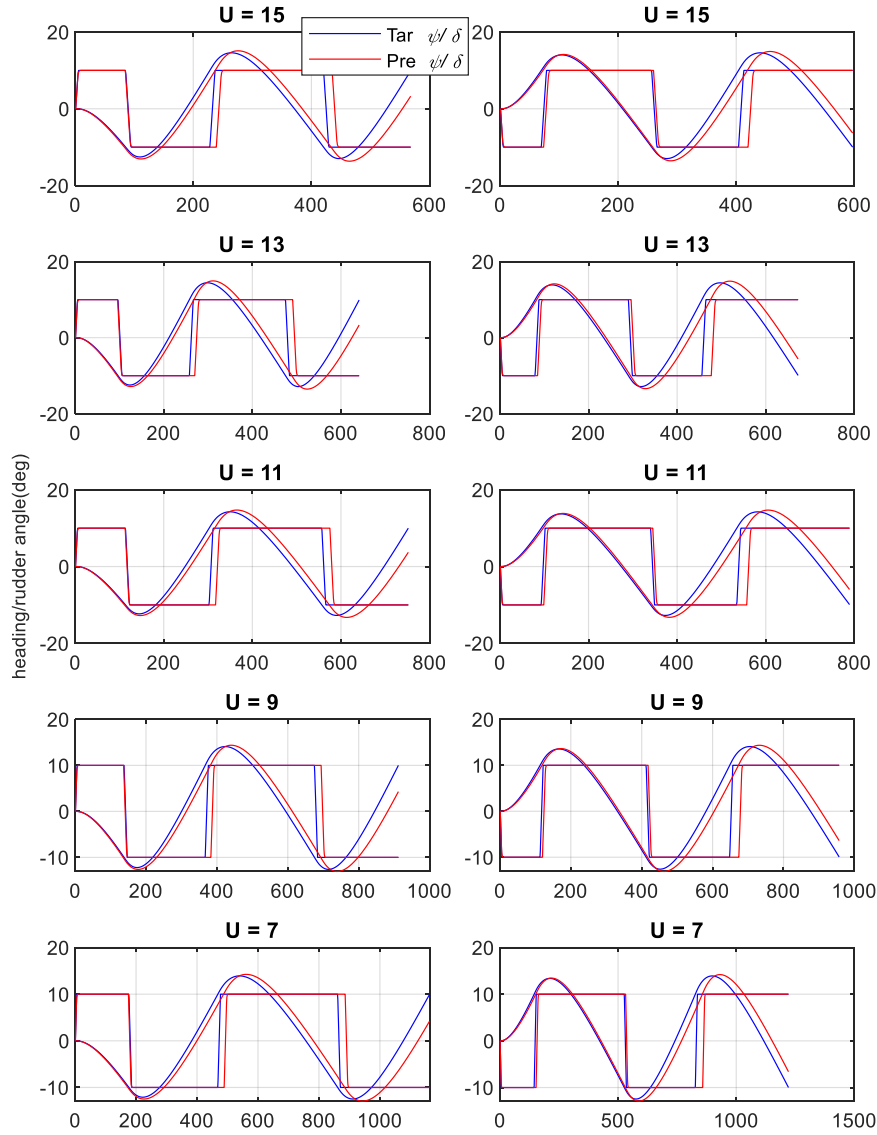


Fig. 6 10°/10° zigzag tests conducted at different speeds for KVLCC2 at water depth $1.25T_m$. Left/ right column: zigzags start to the port/ starboard side

While the model coefficients for the KVLCC2 are identified using simulation data from the FH tabular model, the underlying identification framework is designed for general applicability. The FH model itself is a high-fidelity representation derived from captive physical tests. Utilizing this model as the “target” ensures that the simulated ship response is physically grounded, providing a convincing environment for validating the identification algorithm. It is also important to emphasize that there is a significant structural difference between the FH tabular model (a modular, table-based representation) and the Norrbm model (a parameterized whole-ship polynomial model). The successful parameter identification confirms that the adopted Norrbm model structure is sufficiently flexible to capture and reproduce the underlying ship dynamics across different mathematical formulations. Sea-trial or physical experimental data introduces challenges such as sensor noise and environmental disturbances. Such real-world applications would require appropriate data pre-processing to clean the input-output time series before identification.

3. Motion control

3.1 Model predictive control

He et al. [18] proposed a model predictive controller for ship path following, where a very simple predictive model derived from the Nomoto model [26] was employed. The effectiveness of the controller was

validated through experimental tests around a nominal operating speed. The controller was subsequently extended in [19] to speed-adaptive path following and trajectory tracking at a fixed shallow-water condition. In that study, simplified speed-dependent 3-DOF predictive models were employed, and the prediction horizon was adjusted using scaling laws based on speed, propeller rate, or their combination. The present work retains the general MPC path-following structure but addresses simultaneous variations in water depth and speed. The Norrbin-based manoeuvring model introduced in Section 2.5 replaces the previous speed-dependent predictive model, allowing the prediction dynamics to be updated according to both UKC and vessel speed. The adaptive controller is validated at various water depths and speeds, as described in Section 3.2. Through extensive parameter variation tests, it is found that the essential parameter of MPC, the prediction horizon N_p , is dependent on speed and water depth, while other parameters are not sensitive to changes in operating conditions. The adaptive setting of the prediction horizon is then investigated in Section 3.3. For the sake of proposing a general parameter tuning principle for different ships at various speeds, the prediction horizon is nondimensionalized to:

$$N_p = N'_p \frac{L}{U} \quad (29)$$

where U is ship speed. Detailed mathematical formulations of the MPC are provided in Appendix A.

3.2 Fixed-horizon MPC baselines: path following at different water depths and speeds

At Flanders Hydraulics (FH), real-time full-mission simulators, fast-time simulators and experimental tank facilities are equipped and allow to steer ships based on track controllers. The simulators are driven by accurate shallow and confined manoeuvring models in calm water, which are built through systematic model tests in the FH towing tanks. Several aspects of ship manoeuvring in shallow and confined navigation areas, such as effect of shallow water, bank effects, ship-to-ship interaction, and effect of mud layers, have been investigated by model tests and incorporated in mathematical manoeuvring models at FH [27]. Recently, some models have also started to incorporate disturbances from waves. Provided with a bathymetry, these simulators can replicate complex environments.

Using the API of the FH fast-time simulator [28], the MPC path follower employing the 3-DOF water depth and speed dependent predictive model is tested through extensive simulations. The KVLCC2 benchmark ship in full scale is selected as the control plant. The UKC is varied to 100, 50, 33.33, 25 and 20 % ($\zeta = 1, 2, 3, 4$ and 5), while the ship speed is varied to 7, 9, 11, 13 and 15 knots, creating 25 different operating conditions. The path is planned to generate a S-manoeuve with two counteracting bends (avoiding collision with obstacles around the bends). Because the primary objective is to quantify the controller's adaptability to water depth and speed variations, only the shallow water effects are activated in the simulator (effects of bank, ship-to-ship interaction, mud layers and waves are not included).

Two indices, mean track error integral ($mTEI = \frac{1}{N_e} \sum_{k=k_0}^{k_1} |e(k)|$) and mean rudder integral ($mRI = \frac{1}{N_e} \sum_{k=k_0}^{k_1} |\delta(k)|$), are selected for evaluation of control performance, where e is the cross-track error expressed in the Serret–Frenet frame and $N_e = k_1 - k_0 + 1$ is the number of samples within the evaluation interval. The evaluation is restricted between the first waypoint and the final waypoint. $mTEI$ is reported in metres and mRI in degrees. $mTEI$ identifies a mean deviation of the actual track from the prescribed one, while mRI quantifies the mean rudder deflection. To establish a baseline comparison, a fixed-horizon MPC is evaluated by setting N'_p to a constant, conservative value of 1.5. Figure 7 shows the variation of $mTEI$ and mRI for this baseline controller under different water depth and speed conditions. Very high control accuracy is observed for all 25 operation conditions. All $mTEI$ are respectively less than 3.24 % of the ship's breadth (the absolute maximum track error was maintained below 11.31 % of the ship's breadth). As ζ increases (the UKC decreases), $mTEI$ tends to decrease generally for all speeds (For path following tasks along relatively gentle S-manoeuves, the increased hydrodynamic damping with decreasing UKC acts as a physical stabilizer that naturally suppresses unwanted yawing, drift, and overshoots. Consequently, when the MPC is properly set to handle the initial sluggishness, the highly damped nature contributes to a more stable path following, leading

to a decreased $mTEI$). At higher speeds, $mTEI$ is generally smaller than at lower speeds for the same UKC. This suggests that increasing speed may help reduce $mTEI$ at the same water depth, potentially due to different hydrodynamic effects at varying speeds. It should be noticed that the prediction accuracy, which is influenced by the training data (selection of manoeuvres), may also contribute to the difference in control accuracy at different operating conditions. This is not the focus of the current research, and it is not discussed here. Figure 8 illustrates the tracks and rudder angles for two path following tests operated the extreme conditions. The high control accuracy and efficiency prove the effectiveness of the prediction model and its adaptability to water depth and speed changes. Although the fixed setting maintains stable path following over the complete operating envelope, it is deliberately conservative and does not provide the highest tracking accuracy at every operating condition. Its tracking performance is compared quantitatively with the best tested condition-specific horizons in Section 3.3.

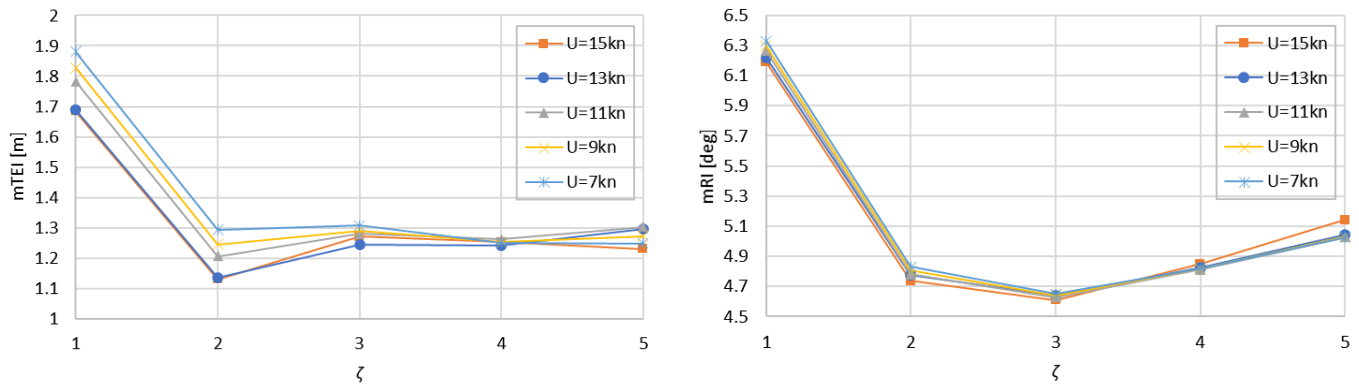


Fig. 7 $mTEI$ and mRI at different water depths and speeds, with N'_p set to 1.5

3.3 Optimal prediction horizon

The prediction horizon is the most essential control parameter in MPC. A larger prediction horizon reduces prediction accuracy – which can jeopardize control accuracy – and results in more moderate control actions, and vice versa. However, a very small prediction horizon may lead to very aggressive control actions and then cause the system to become out of control, which should be avoided in practice. These were observed at various speeds for a 1/70 scale model of a cape-size bulk carrier with a length of around 4m in shallow water (25 % UKC) and elaborated in [29].

With a conservative constant N'_p , accurate path following can be achieved at different speeds and water depths. But it is still valuable to explore the optimal prediction horizon for further improvement in control accuracy and to check the limits of setting this parameter. Firstly, a large resolution of N'_p (0.25) is set to explore the limits and N'_p is varied to: 0.75, 1, 1.25, 1.5, 1.75, 2. Figure 9 plots the $mTEI$ varying with N'_p at the deepest water and the shallowest water. It demonstrates that the highest control accuracy occurs with the same setting of N'_p for different speeds (the optimal N'_p). However, it decreases with water depth. This implies that water depth significantly influences the setting of N'_p but the speed does not. It should be noticed that at a higher speed, the controller is much more sensitive to the setting of N'_p : reducing the optimal N'_p will noticeably increase tracking error and very likely cause the system to become out of control. The optimal N'_p is therefore also the minimal one to ensure the stability of the control.

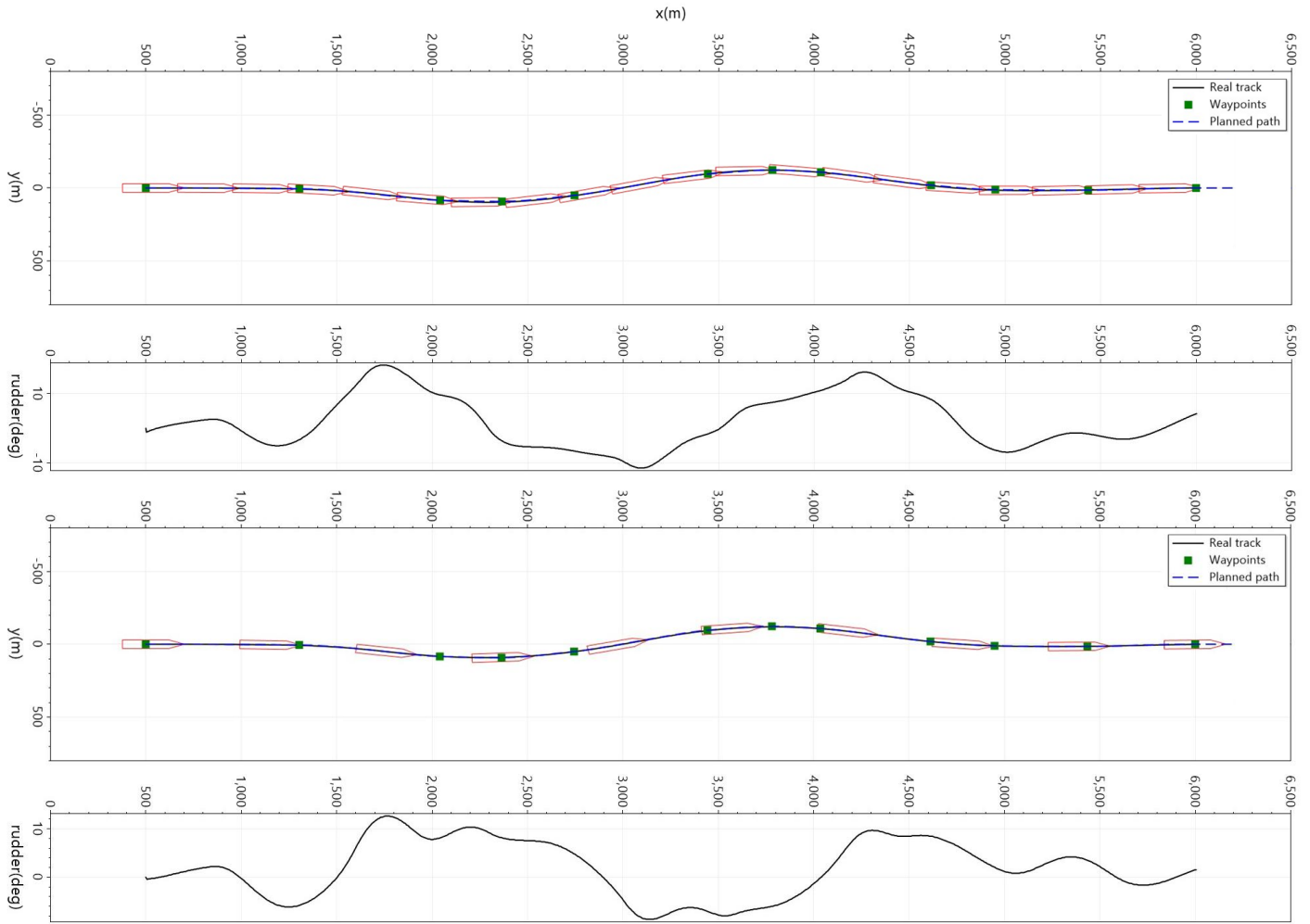


Fig. 8 Trajectories and control actions in two path following tests. Top two plots: at the largest water depth and the lowest speed; Other plots: at smallest water depth and the highest speed

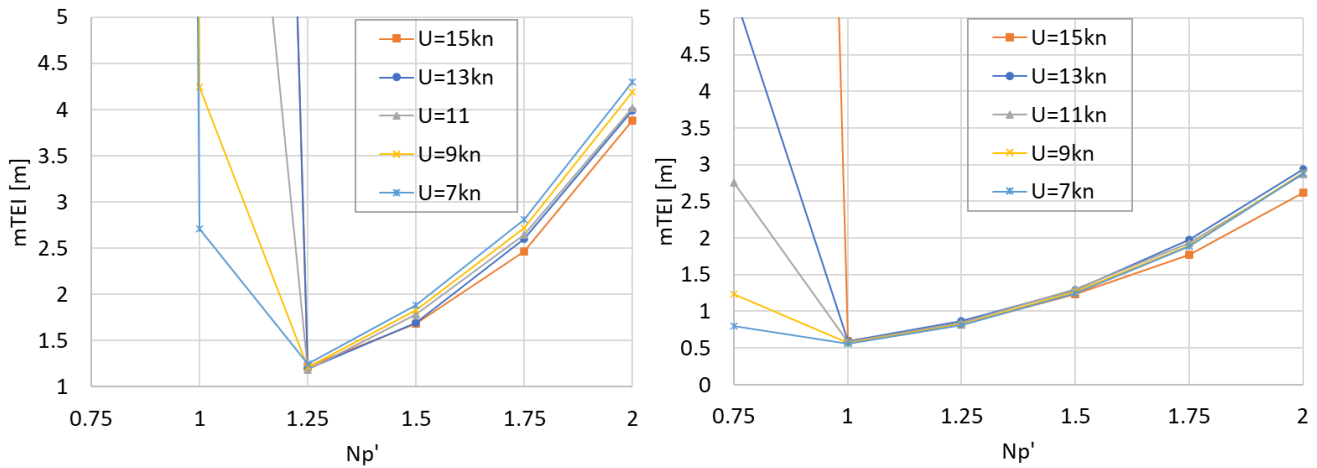


Fig. 9 $mTEI$ varying with N_p' at various speeds. Left plot: $\zeta = 1$; Right plot: $\zeta = 5$

A high-resolution parameter setting grid is then defined to accurately locate the optimal N_p' : $N_p' \in [0.75, 1.25]$ with resolution of 0.05. Figure 10 shows the $mTEI$ when varying N_p' at different speeds at a constant water depth ($\zeta = 5$), and it indicates that the optimal horizon actually slightly increases with the speed. Figure 11 plots all the optimal N_p' at various water depths and speeds and confirms the fact that the optimal

N'_p ($N'_{p,opt}$) significantly increases with water depths and slightly increases with speed. Similar linear trends are observed at each water depth and speed in the plot:

$$N'_{p,opt} = N'_{pH} + \frac{N'_{pH} - N'_{pL}}{U_H - U_L} (U - U_H) \quad (30)$$

$$N'_{p,opt} = N'_{pD} + \frac{N'_{pD} - N'_{pS}}{\zeta_D - \zeta_S} (\zeta - \zeta_D) \quad (31)$$

where subscripts 'H', 'L', 'D' and 'S' respectively represent the highest speed, the lowest speed, the deepest water and the shallowest water. Based on these observations a general formula can be obtained for estimating the optimal N'_p :

$$\begin{aligned} N'_{p,opt} &= N'_{p,DH} + k_U (U - U_H) + k_\zeta (\zeta - \zeta_D) \\ k_U &= \min \left(\frac{N'_{p,DH} - N'_{p,DL}}{U_H - U_L}, \frac{N'_{p,SH} - N'_{p,SL}}{U_H - U_L} \right) \\ k_\zeta &= \max \left(\frac{N'_{p,DH} - N'_{p,SH}}{\zeta_D - \zeta_S}, \frac{N'_{p,DL} - N'_{p,SL}}{\zeta_D - \zeta_S} \right) \end{aligned} \quad (32)$$

where the smaller slopes of $N'_{p,opt}$ with regard to water depth or speed (absolute values) are adopted since a larger estimation of $N'_{p,opt}$ is more conservative (safer). Because the closed-loop response is highly asymmetric around the best-tested prediction horizon, underestimating N'_p is substantially more critical than overestimating it. In particular, Figures 9 and 10 shows that a small decrease in N'_p below the best-tested value may move the controller rapidly into a high-sensitivity region, especially at high speeds. Therefore, the prediction horizon estimated from the manoeuvring index is treated as a nominal value rather than being applied directly. Following Equation (32), the prediction horizon actually used by the controller is shifted toward the conservative side by a positive stability margin $\Delta N'_p$:

$$N'_p = N'_{p,opt} + \Delta N'_p \quad (33)$$

A recommended setting of $\Delta N'_p$ is from 0.05 to 0.1. The margin is introduced as an engineering safeguard against the high sensitivity associated with horizon underestimation and is not intended as a formal stability guarantee.

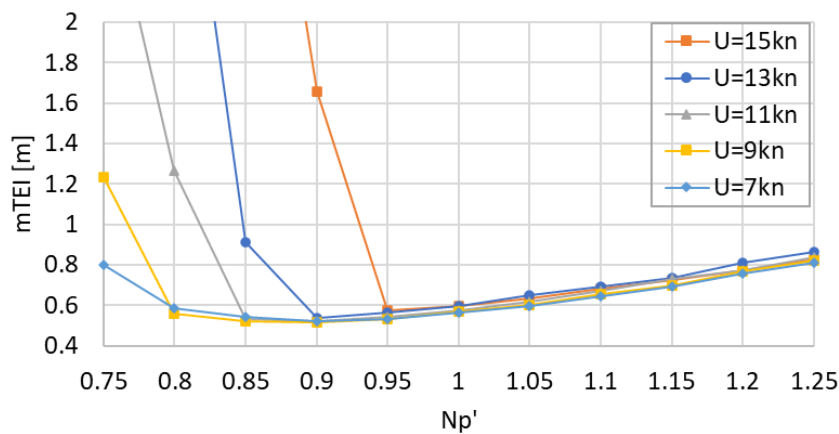


Fig. 10 $mTEI$ varying with N'_p at various speeds, with $\zeta = 5$

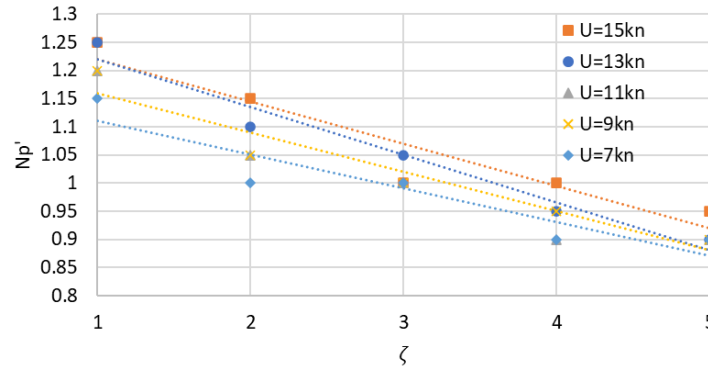


Fig. 11 The optimal N'_p varying with water depth at different speeds

Table 5 Comparison of the fixed, proposed adaptive, and best-tested prediction horizons over the 25 operating conditions

ζ	Speed (kn)	Fixed horizon		Best tested horizon			$mTEI$ reduction (%)	T'_K	Practical estimates with Equations (33) and (36)	
		$mTEI$ (m)	mRI (deg)	N'_p	$mTEI$ (m)	mRI (deg)			N'_p	Interpolated $mTEI$ (m)
5	15	1.23	5.14	0.95	0.58	6.53	53.11	1.01	1.02	0.61
5	13	1.30	5.05	0.9	0.54	6.79	58.52	0.97	0.99	0.59
5	11	1.30	5.03	0.9	0.52	6.54	60.07	0.98	0.99	0.58
5	9	1.27	5.03	0.9	0.52	6.43	59.28	0.98	1.00	0.57
5	7	1.25	5.03	0.9	0.52	6.38	58.15	0.98	1.00	0.56
4	15	1.25	4.85	1	0.64	6.15	49.01	1.05	1.06	0.67
4	13	1.24	4.83	0.95	0.54	6.60	56.65	1.03	1.04	0.63
4	11	1.26	4.81	0.9	0.53	6.98	58.15	1.03	1.05	0.58
4	9	1.25	4.82	0.95	0.49	6.01	60.88	1.04	1.05	0.56
4	7	1.25	4.82	0.9	0.46	6.29	63.18	1.04	1.05	0.55
3	15	1.27	4.61	1	0.71	6.41	44.14	1.10	1.11	0.82
3	13	1.24	4.64	1.05	0.75	5.53	39.40	1.10	1.11	0.77
3	11	1.28	4.63	1	0.69	5.94	46.02	1.10	1.11	0.73
3	9	1.29	4.64	1	0.65	5.77	49.59	1.10	1.11	0.70
3	7	1.31	4.65	1	0.62	5.75	52.95	1.10	1.11	0.69
2	15	1.13	4.74	1.15	0.76	5.70	33.10	1.19	1.19	0.77
2	13	1.14	4.78	1.1	0.70	6.15	38.46	1.18	1.18	0.75
2	11	1.21	4.78	1.05	0.67	6.34	44.19	1.18	1.18	0.74
2	9	1.25	4.81	1.05	0.64	6.25	48.33	1.18	1.18	0.74
2	7	1.29	4.83	1	0.63	6.51	51.31	1.18	1.18	0.75
1	15	1.68	6.19	1.25	1.22	6.44	27.40	1.33	1.31	1.27
1	13	1.69	6.22	1.25	1.19	6.40	29.43	1.32	1.31	1.27
1	11	1.78	6.26	1.2	1.18	6.63	33.91	1.31	1.30	1.26
1	9	1.83	6.29	1.2	1.17	6.56	36.20	1.31	1.30	1.31
1	7	1.88	6.33	1.15	1.18	6.79	37.36	1.30	1.29	1.33

Note: The best tested N'_p is defined as the stable value among the explicitly evaluated horizons that yields the minimum $mTEI$. The $mTEI$ reduction is calculated relative to the fixed-horizon baseline $N'_p = 1.5$. The practical adaptive horizon is calculated as $N'_p = 0.915T'_K + \Delta N'_p$, and their corresponding $mTEI$ values are obtained by piecewise-linear interpolation between neighboring simulated results.

Table 5 demonstrates that the benefit of condition-specific horizon selection is substantial rather than marginal. For the 25 conditions with complete numerical comparisons, $mTEI$ is reduced under every condition, with reductions ranging from 27.40 to 63.18 % and an average condition-wise reduction of 47.55 %.

3.4 Self-tuning prediction horizon

In practice, determining the optimal N'_p at a certain operating condition is still difficult since a series of parameter variation tests need to be done. A good estimation of a proper value can help reduce the tuning time. [18] proposed to use $\frac{1}{K}$, T or $\frac{L}{U}$ to obtain an initial guess, since they all have time units and can reflect the ship dynamic changes. K and T are the parameters of the Nomoto model [26]. The $\frac{L}{U}$ coefficient is well applicable when speed changes at the same water depth but cannot reflect the dynamic changes induced by water depth. Additionally, $\frac{L}{U}$ can be used for nondimensionalization of variables with time units to exclude the effects of ship speed and scale. Another parameter, $T_K = \sqrt{\frac{T}{|K|}}$, which includes both K and T , is proposed for consideration (The smaller T_K , the more manoeuvrable). The three candidates, $\frac{1}{K}$, T and T_K , are normalized by:

$$K' = K \frac{L}{U}, T' = T \frac{U}{L}, T'_K = T_K \frac{U}{L} \quad (34)$$

Figure 12 plots K' , T' and T'_K with the water depth variable ζ at various speeds, where the Nomoto model parameters are estimated by using the linear least square method with data from multiple zigzags. As seen, $\frac{1}{K'}$ noticeably decreases with water depth, T' slightly decreases with speed, and T'_K increases with water depth noticeably and speed slightly. Therefore, $\frac{1}{K'}$ and T' cannot be adopted to estimate $N'_{p,opt}$ since they do not share the same trends when speed and water depth change. Furthermore, the increasing trend of T'_K with water depth is closely linear, which is similar to that of $N'_{p,opt}$, and its value at each water depth and speed is also slightly larger than $N'_{p,opt}$, indicating T'_K is a highly appropriate variable to estimate $N'_{p,opt}$ at each operating condition. The average trendline of $N'_{p,opt}$ and T'_K are (average slope and intercept at different speeds):

$$\begin{aligned} N'_{p,opt} &= -0.072\zeta + 1.246 \\ T'_K &= -0.080\zeta + 1.365 \end{aligned} \quad (35)$$

As shown in Figure 13, a proportional regression between the best-tested prediction horizon and the combined manoeuvring index T'_K gives:

$$N'_{p,opt} \approx K_p T'_K, K_p = 0.915 \quad (36)$$

with $R^2 = 0.9186$.

If ignoring the small effect due to speed change, Equation (35) can be utilized for estimation of $N'_{p,opt}$ at each water depth. But the linear relationship may only be valid for ships with a similar ship type (tanker) and actuator setting (single rudder and single propeller). For performance improvement, T'_K should be identified by manoeuvring tests and then $N'_{p,opt}$ can be determined with Equation (36). Compared with the speed-only scaling rules in [19], Equation (36) uses the manoeuvring index T'_K , which varies with both water depth and speed, thereby extending the tuning principle of prediction horizon to the two-dimensional operating envelope considered in this study. As demonstrated by the horizon-sensitivity results in Figure 10, a small underestimation of N'_p may cause a disproportionately large deterioration in tracking performance. Therefore, the nominal estimate from Equation (36) is combined with the positive margin introduced in Equation (33) to schedule practical prediction horizon. In the present study, $\Delta N'_p = 0.1$ is adopted. For the 25 investigated operating conditions, the maximum one-sided underestimation of the nominal estimate relative to the best-tested horizon is approximately 0.061. Hence, the adopted margin places all scheduled horizons on the

conservative side of the corresponding best-tested values. Figure 13 illustrates that the practical estimates are all larger than the best-tested horizon (the boundary between the sensitive region and the conservative region).

Table 5 distinguishes explicitly among the fixed horizon, the horizon generated by the proposed practical scheduling law, and the retrospectively determined best-tested horizon. Because the practically scheduled horizons generally fall between the discrete values used in the existing closed-loop horizon sweep, their corresponding $mTEI$ values are obtained by piecewise-linear interpolation between neighbouring simulated results. The proposed practical scheduler delivers results close to the best-tested horizon (as shown in Figure 14) and improves $mTEI$ relative to the fixed setting under all 25 investigated operating conditions. The expected condition-wise reductions range from 24.64 to 56.20 %, with an average reduction of 42.63 %. The mean $mTEI$ decreases from 1.355 m for the fixed-horizon controller to 0.791 m for the practical adaptive setting. The best-tested horizon remains superior, as expected, because it is selected retrospectively from the parameter sweep to minimize $mTEI$. It is therefore used only as a reference benchmark. The proposed scheduler deliberately operates on the larger- N_p' side of this benchmark by adding the positive margin $\Delta N_p' = 0.1$. This sacrifices a small amount of tracking performance relative to the retrospective optimum but avoids the steep high-sensitivity region associated with horizon underestimation, as illustrated in Figure 14.

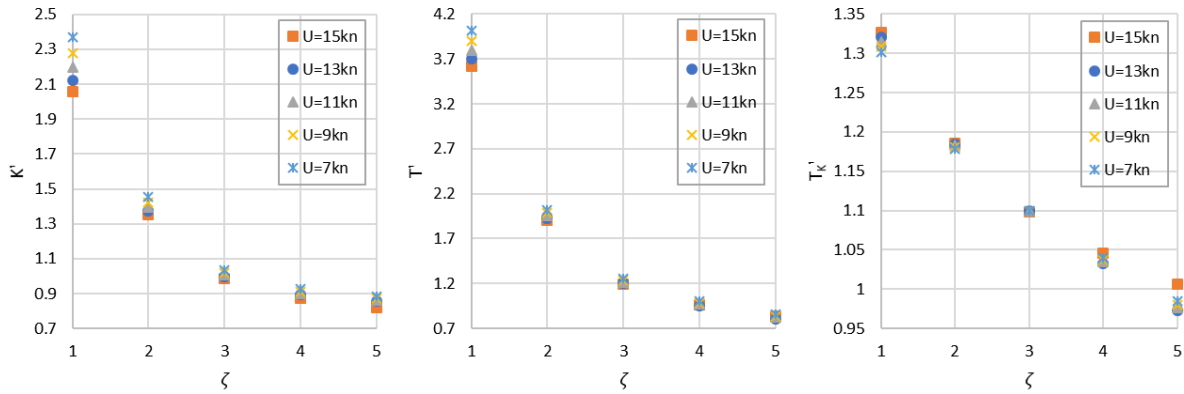


Fig. 12 K' , T' and T'_K identified by using a dataset consisting of multiple zigzags

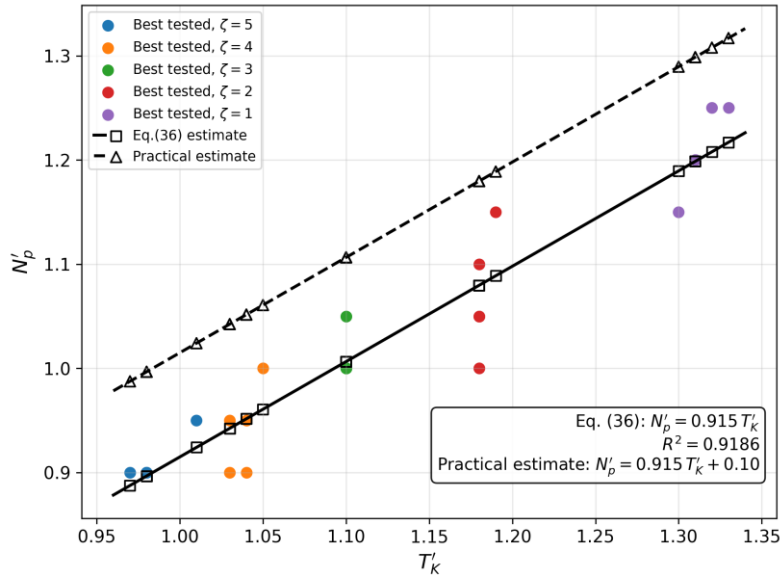


Fig. 13 Practical estimate of nondimensional prediction horizon

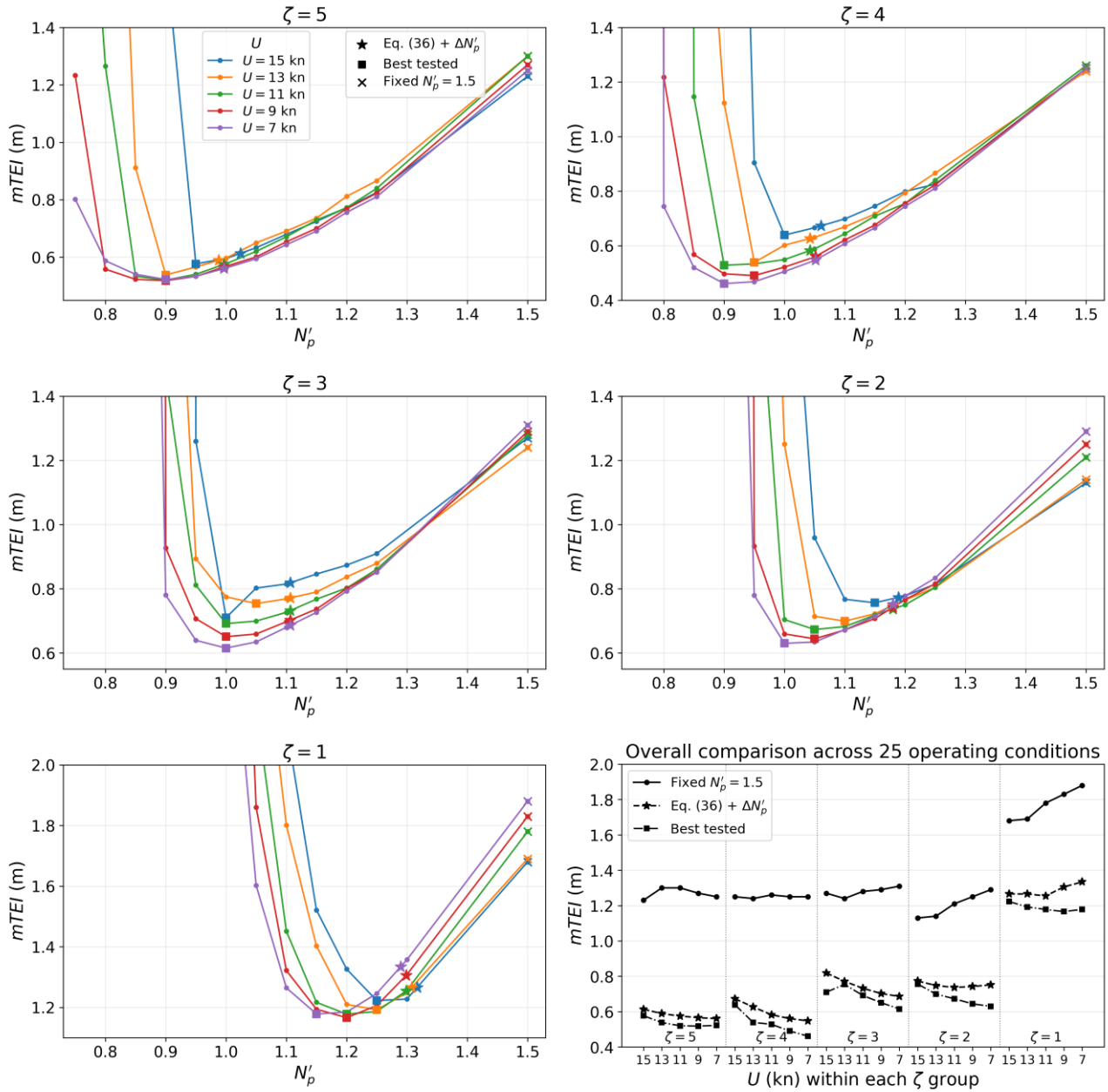


Fig. 14 Comparison of control accuracy among fixed horizon, best-tested horizon and practically estimated horizon

The estimation of K' and T' is very sensitive to the input-output data. As seen in Table 6, at the same water depth, very different parameter values are obtained when using various input-output datasets. However, the estimation of T'_K is less sensitive to the selection of the input-output dataset, and similar values are calculated. This also confirms the preference to use T'_K for estimating a suitable N'_p . For practical application, the coefficients in Equations (35) and (36) can be obtained offline with manoeuvring tests conducted at several water depths (at least two). It is suggested to use a combination of zigzags for system identification, which can maintain higher estimation accuracy. To execute the proposed adaptive prediction horizon in real-time control, the tuning logic should update with the same frequency as the MPC sampling rate.

Table 6 Parameter estimation of K' , T' and T'_K with different manoeuvres at various water depths at 15kn (naming code of manoeuvres: 'Z'=zigzag, 'xx'=maximum rudder angle, 'xx'=switching heading, 'S/P'=starting to the starboard/port)

ζ	Combination			Z1005S			Z1010P			Z1010S		
	K'	T'	T'_K	K'	T'	T'_K	K'	T'	T'_K	K'	T'	T'_K
1	-2.06	3.62	1.33	-6.23	9.80	1.25	-4.05	6.03	1.22	-4.09	6.54	1.27
2	-1.35	1.90	1.19	-1.84	2.28	1.11	-1.92	2.26	1.09	-1.87	2.34	1.12
3	-0.99	1.19	1.10	-1.12	1.17	1.02	-1.16	1.17	1.01	-1.13	1.20	1.03
4	-0.87	0.96	1.05	-1.01	0.96	0.98	-1.02	0.93	0.96	-0.99	0.96	0.98
5	-0.82	0.83	1.01	-0.97	0.89	0.96	-0.97	0.84	0.93	-0.93	0.87	0.96
ζ	Z1505S			Z2010P			Z3005S			Z3505S		
	K'	T'	T'_K	K'	T'	T'_K	K'	T'	T'_K	K'	T'	T'_K
1	-3.48	6.07	1.32	-1.77	2.73	1.24	-1.42	2.44	1.31	-1.24	2.38	1.39
2	-1.64	2.20	1.16	-1.36	1.70	1.12	-1.14	1.62	1.19	-1.03	1.64	1.26
3	-1.05	1.18	1.06	-1.03	1.13	1.04	-0.91	1.11	1.10	-0.83	1.14	1.17
4	-0.93	0.96	1.01	-0.91	0.89	0.99	-0.82	0.89	1.05	-0.75	0.92	1.11
5	-0.88	0.86	0.99	-0.84	0.76	0.95	-0.76	0.77	1.00	-0.70	0.78	1.06

To further prove the effectiveness of the findings, analysis of experimental results will be provided. In [28], the MPC was extensively tested with a scale model of a cape-size bulk carrier in the Towing Tank for Manoeuvres in Confined Water, Antwerp, Belgium [18]. Varying N'_p to 0.5, 0.75, 1, 1.25 and 1.5 at 25% UKC ($\zeta = 4$), the best setting for N'_p was found to be around 1, with $N'_p = 0.5$ providing very bad performance and $N'_p > 1$ delivering satisfactory performance (the trend is similar to that shown in Figure 9). Table 7 lists the identified T'_K for the bulk carrier at various speeds, and it is seen that the estimation of $N'_{p,opt}$ by using Equation (36) is well-matched with experimental determination. Additionally, T'_K and $N'_{p,opt}$ estimated by using Equation (35) is 1.04 and 0.96, which are also very close to identified T'_K and the best N'_p determined from experimental tests.

The formulation of Equations (32)-(36) is intended as a practical engineering guideline derived from extensive sensitivity analysis. While the linear relationships are empirical, the choice of the T'_K index as the primary estimator is rooted in ship dynamics theory, as it effectively represents the vessel's manoeuvrability in shallow water. The effectiveness of this heuristic is further supported by its successful cross-validation with experimental results from different hull forms as introduced above. Furthermore, the practical implementation of the adaptive MPC mechanism is shown in Table 8. The proposed scheduling law is intended to operate within the physically admissible operating envelope of a specific vessel rather than to perform unrestricted extrapolation. For a given ship, the practical cruising-speed range is bounded by propulsion and operational constraints, i.e., $U_{min} \leq U \leq U_{max}$. Similarly, the minimum allowable UKC (UKC_{min}) is normally prescribed by navigational safety requirements and local regulations. The physically relevant range of ζ is then bounded by $0 < \zeta \leq 1/UKC_{min}$. Therefore, the offline manoeuvring tests used to establish Equations (35) and (36) should be selected to cover the intended operational envelope of the vessel. The proposed tuning relationship is applied within this predefined range, and unrestricted extrapolation beyond the vessel's admissible speed or UKC range is neither required nor claimed.

It is essential to emphasize that the adaptive model and the adaptive horizon are not independent algorithmic add-ons; rather, they are hydrodynamically coupled mechanisms responding to the same physical perturbation, the variation in water depth and speed. Specifically, the variation directly alters the ship's hydrodynamics (necessitating model adaptation) while concurrently shifting the response characteristics of the vessel (necessitating horizon adaptation to match the updated system dynamics). Separating these two mechanisms would yield an ill-posed control problem: it would force the controller to either accumulate massive prediction errors by applying an adaptive horizon to a static, mismatched hydrodynamic model, or risk severe control instability by applying a fixed prediction horizon to a system with fundamentally changed dynamics. Thus, they collectively constitute an inseparable, integrated adaptation framework.

Table 7 T'_K and N'_p for the cape-size bulk carrier (length: 4.029m, breadth: 0.643m, draft: 0.236m) at 25% UKC

U [m/s]	T_k [s]	T'_K [-]	Best N'_p [-]	Estimated N'_p with Equation (36) [-]
0.18	24.99	1.12	{0.75,1}	1.01
0.31	14.65	1.13	{0.75,1}	1.02
0.43	10.29	1.10	{0.75,1}	0.99
0.55	8.19	1.12	{1,1.25}	1.01
0.68	6.62	1.12	{1,1.25}	1.01

Table 8 Pseudocode of the adaptive MPC mechanism

Algorithm 1: Adaptive MPC-based Track Control with Water Depth and Speed Variations
Phase 1: Offline Preparation 1: Define operating conditions (different water depths and speeds) 2: Collect navigation data by performing zigzag maneuvers at several water depths and speeds 3: Identify the water depth and speed dependent predictive model represented by Equations (21), (22), (25) and (27) 4: Identify the Nomoto parameters K and T at tested water depths and speeds, calculate the corresponding T'_K 5: Take the average value of T'_K over speed at the same water depth and interpolate T'_K for other water depths using linear fitting as represented by Equation (35)
Phase 2: Online Procedure 6: for every sampling instant k do : 7: Acquire current ship states (e.g. speed) and actual water depth h_k 8: Compute the value of the water depth parameter ζ_k by Equation (4) 9: Update the non-dimensional prediction horizon N'_p using the empirical formula Equations (33) and (36), and apply the prescribed bounds to N'_p , [0.75, 2], if required. 10: Convert N'_p to the dimensional prediction horizon N_p (converted into an integer number of prediction steps by rounding) 11: Update the predictive model using current water depth via Equations (21), (22), (25), and (27) 12: Return N_p and the updated predictive model to the MPC optimizer to predict ship motion and solve for the optimal control sequence 13: end for

4. Conclusion

This study extends the authors' previous speed-adaptive MPC approach, which was developed and experimentally validated at a fixed shallow water condition [19], to operating conditions involving simultaneous variations in water depth and vessel speed. A Norrbinn model based predictive motion controller is developed for surface vessels actuated by conventional propellers and rudders, assuming self-propulsion propeller rates. The least square method is applied for parameter estimation of the manoeuvring model, based on simulated free running test data. The identified model is validated by comparing its predictions with the target responses generated by the high-fidelity simulator for standard manoeuvres at various operating conditions. Employing the identified model to MPC path following delivers very high accuracy at each operating condition. Specifically, the validation across 25 combinations of depth and speed (UKC 20-100 %

and speeds 7-15 knots) demonstrates that the controller maintains the $mTEI$ below 3.24 % of the ship's breadth when using the conservative fixed horizon $N'_p = 1.5$. Despite the substantial structural difference between the FH tabular plant model and the polynomial Norrbins predictive model, satisfactory prediction and control performance is obtained over the investigated operating envelope. Tests at water depth and speed conditions beyond those directly represented in the identification dataset further demonstrate the model's interpolation/extrapolation capability and its generalization across operating conditions.

However, the prediction-horizon sweep shows that the fixed setting is unnecessarily conservative. Selecting the best tested condition-specific horizon reduces $mTEI$ by 27.40-63.18 %, with an average reduction of 47.55 % over the 25 operating conditions. Therefore, to further improve the adaptability of the MPC path follower, a self-tuning principle for the prediction horizon is then investigated. A practical estimation relationship based on the combined manoeuvring index T'_K is proposed, enabling the prediction horizon to be adjusted according to the current water depth and vessel speed. The resulting adaptive horizons remain on the conservative side of the best-tested values throughout the investigated operating envelope and consistently improve tracking performance relative to the fixed-horizon baseline.

To summarize, the track controller's adaptability to water depth and speed change has been realized by incorporating a water depth and speed dependent predictive model into MPC and implementing self-tuning prediction horizon. For future extensions, this adaptive framework can be applied to waterways with spatially varying bathymetry. By integrating real-time depth sensing from onboard echo sounders or digital bathymetric maps, the controller can dynamically update the UKC parameter. This allows the predictive model and the self-tuning prediction horizon to adapt continuously to the changing environment as the vessel moves along the route. A solid validation should also involve experimental testing, such as in towing tanks. Furthermore, robustness against external disturbances, measurement noise, model parameter uncertainties, and water depth estimation errors was not quantitatively evaluated in this study. These aspects require dedicated perturbation tests and remain subjects of future work.

Nomenclature

Symbol	Description	Unit
x, y, ψ	Ship position and heading in the earth-fixed frame	m, rad
u, v, r	Surge velocity, sway velocity, and yaw rate in the body-fixed frame	m/s, rad/s
L, B, T_m	Ship length, beam, and draft	m
h	Water depth	m
UKC, ζ	Under keel clearance and water depth parameter ($1/UKC$)	-
δ, n	Rudder angle and propeller rate	deg, rpm
β, γ	Drift angle and yaw rate angle	rad
X, Y, N	Hydrodynamic forces and moment in surge, sway, and yaw	N, N·m
K, T	Nomoto model gain and time constant	1/s, s
T_K	Combined manoeuvrability index	s
U	Ship speed	m/s
$N_p, \Delta N'_p$	Prediction horizon, stability margin of prediction horizon	s, -
$mTEI$	Mean track error integral in the Serret-Frenet frame	m
mRI	Mean rudder angle integral	deg

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Appendix A: Mathematical formulation of the model predictive controller

A.1 State vector and reference trajectory. The path following problem is formulated based on the Serret-Frenet (SF) frame moving along the planned path. The ship's state vector at the i -th prediction step is defined in the horizontal plane as:

$$\mathbf{x}_i = [x_i, y_i, \chi_i, u_i, v_i, r_i]^T \quad (37)$$

where χ represents the predicted course. The corresponding reference state vector at the i -th step is sampled from the planned parametric trajectory based on the path variable ϖ :

$$\mathbf{x}_i^{\text{ref}} = [p_x(\varpi_i), p_y(\varpi_i), p_\chi(\varpi_i), p_u(\varpi_i), p_v(\varpi_i), p_r(\varpi_i)]^T \quad (38)$$

where p_x and p_y are the reference positions, p_χ is the path tangential direction, and p_u, p_v, p_r are reference velocities. The prescribed waypoints are connected using piecewise cubic Hermite interpolation (PCHIP) to construct a continuous parametric path $\mathbf{p}(\varpi) = [p_x(\varpi), p_y(\varpi)]^T$. The current reference point is represented by the virtual target of the moving Serret-Frenet frame. Its path variable is advanced according to

$$\dot{\varpi} = \frac{U \cos \chi_{SF} + K_2 s}{\sqrt{p'_x(\varpi)^2 + p'_y(\varpi)^2}} \quad (39)$$

where s and χ_{SF} are the along-track error and the relative course in the Serret-Frenet frame and K_2 is the convergence factor (fixed at 0.1 in this paper).

Starting from the current path variable ϖ_0 , the reference points over the MPC prediction horizon are sampled directly from the PCHIP path. Since no additional reference-path transition is employed in the present simulations, the path variable is recursively advanced as

$$\varpi_{i+1} = \varpi_i + \frac{U_d t_s}{\sqrt{p'_x(\varpi_i)^2 + p'_y(\varpi_i)^2}} \quad (40)$$

where U_d is the desired speed, and t_s is the sampling time ($t_s = 1$ s). For path-following operation, the reference propagation uses the current speed ($U_d = U$).

A.2 Objective function. The core objective of the MPC is to minimize the deviation between the predicted ship states and the reference trajectory over a predefined prediction horizon (N_p). For path following applications where the propeller rate remains constant, the control input vector simplifies to the commanded rudder angle δ_c . The optimization problem is formulated as a quadratic cost function:

$$\text{argmin } J(\delta_c) = \sum_{i=1}^{\text{round}(N_p/t_s)} (\mathbf{x}_i - \mathbf{x}_i^{\text{ref}})^T \mathbf{Q} (\mathbf{x}_i - \mathbf{x}_i^{\text{ref}}) \quad (41)$$

To penalize the cross-track error and heading error, the positive semi-definite weighting matrix is specifically designed as:

$$\mathbf{Q} = \text{diag} \left[1, 1, w_H \frac{180}{\pi}, 0, 0, 0 \right] \quad (42)$$

where w_H represents the heading weight factor, and it is set at 0.1 in all simulations. The factor $\frac{180}{\pi}$ follows the implementation convention for weighting the course error expressed in radians. The position error weights are both set to unity, while the velocity states are unpenalized for path following tasks.

A.3 System dynamics and actuator constraints. The predictive model relies on the discrete-time system dynamics updated via the Euler method with a constant sampling time t_s :

$$\mathbf{x}_{i+1} = \mathbf{x}_i + f(\mathbf{x}_i, \delta_i)t_s \quad (43)$$

where $f(\cdot)$ is the nonlinear vector function describing the 3-DOF manoeuvring dynamics incorporating the shallow water effects. Crucially, to prevent unfeasible control commands, the physical limits of the steering gear are explicitly enforced as hard constraints within the predictive model. The actuator constraints include the rudder saturation (maximum rudder angle $\delta_{\max}$) and the steering rate limit (maximum rudder rate $\dot{\delta}_{\max}$):

$$\begin{aligned} \delta_c &= \text{sign}(\delta_c) \min(|\delta_c|, \delta_{\max}) \\ \dot{\delta}_i &= \text{sign}(\delta_c - \delta_{i-1}) \min\left(\frac{|\delta_c - \delta_{i-1}|}{t_s}, \dot{\delta}_{\max}\right) \\ \delta_i &= \delta_{i-1} + \dot{\delta}_i t_s \end{aligned} \quad (44)$$

In this paper, $\delta_{\max} = 35$ degrees, and $\dot{\delta}_{\max} = 2.32$ deg/s.

A.4 Numerical solver and implementation. At each sampling instant, the constrained nonlinear optimization problem is efficiently solved online using the COBYLA (Constrained Optimization by Linear Approximation) algorithm. This derivative-free solver is highly suitable for handling nonlinear manoeuvring models and hard actuator constraints. Once the solver determines the optimal control sequence for the entire prediction horizon, only the control actions within the control horizon is applied to the ship's actuators. The horizon window then shifts forward, and the optimization is repeated with the newly updated ship states. In this paper, $N_c = t_s$, and the initial and final trust-region radii of COBYLA are 0.5 and 10^{-6} , respectively. The maximum number of function evaluations is limited to 20.

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